PROJECT: LEVEL CURVES OF HARMONIC POLYNOMIALS

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Motivation. The level sets of harmonic functions and eigenfunctions of the Laplace-Beltrami operator attract a lot of attention of pure and applied mathematicians. Recently, new results appeared and the interest is increasing. At the same time some elementary questions remain open. In this project a question of quantitative approximation of a closed curve by level sets of harmonic polynomial will be studied.

Problem. The maximum principle implies that any level set of a harmonic function defined on the whole plane does not contain any closed curves. The question is how one can approximate for example a circle by (a part of) a level set of a harmonic polynomial and to measure the quality of the approximation when the degree of the polynomial is given.

Details. It follows from a version of Runge's approximation theorem that approximation described above is possible (some details are written down in a recent article by A. Enciso, Alberto and D. Peralta-Salas, Some geometric conjectures in harmonic function theory. Ann. Mat. Pura Appl. (4) 192 (2013), no. 1, 4959) . The quantitative dependence of the approximation of the degree of polynomial is however not known. In the project some specific harmonic polynomials, so called Fejer polynomials and their versions, will be considered. There are numerical evidence that the level sets of such polynomials come close to a circle (without a small arc). This example would give an estimate on the rate of approximation from above. Detailed analysis of the example, optimization of parameters and careful estimates should be done. A theoretical bound for the rate from below can be also studied.

Prerequisites and workload. A good understanding of basic courses including Mathematics 4K is required, a course in Fourier analysis is not required but can give more insight in the problem. Some experience with MATLAB would be an advantage. Other necessary training will be provided. The expected workload is about 80 hours.