Norwegian University of Science and Technology
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MA0301 Elementary discrete mathematics Spring 2018

Solutions to exercise set 4

1 **15.1.1c)** $wx + \overline{y} + yz = 1 \cdot 1 + \overline{0} + 0 \cdot 0 = 1 + 1 + 0 = 1$

d)

$$\begin{aligned}(wx + y\overline{z}) + w\overline{y} + \overline{(w+y)(\overline{x}+y)} &= (1 \cdot 1 + 0 \cdot \overline{0}) + 1 \cdot \overline{0} + \overline{(1+0)(\overline{1}+0)} \\ &= (1+0) + 1 + \overline{1 \cdot 0} \\ &= 1\end{aligned}$$

15.1.2a) $x + xy + w = 1 + 1y + w = 1$

b) $xy + w = y + w$. All assignments of values except $y = w = 0$ give the expression the value 1. $4 - 1 = 3$ in total.

2 **15.1.11a)**

$$\begin{aligned}xy + (x+y)\overline{z} + y &= xy + x\overline{z} + y\overline{z} + y \\ &= x\overline{z} + (x + \overline{z} + 1)y \\ &= x\overline{z} + 1 \cdot y \\ &= x\overline{z} + y\end{aligned}$$

15.1.12) We see that x must be 0, because otherwise the left side of the first equation would be 1. This means we can simplify the first equation to $0 + \overline{0} \cdot y = y = 0$ and the second to $\overline{0} \cdot y = \overline{0} \cdot z \implies z = y = 0$. Putting all this into the third equation gives $\overline{0} \cdot 0 + \overline{0} \cdot \overline{0} + 0w = \overline{0} \cdot w \Leftrightarrow 1 = w$. Thus the only possible solution is $(x, y, z, w) = (0, 0, 0, 1)$, and this satisfies all the equations.

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i) $(x + \overline{y}) \cdot (x + \overline{z}) \cdot (y + \overline{x})$

ii) $(x + y + \overline{z}) \cdot (x + \overline{y} + z)$

iii) $x + y + x \cdot 1 \cdot (z + 0)$

4

i) $xy\overline{x}z = (x\overline{x})yz = 0yz = 0$ (Inverse law)

ii) $xyzy = x(yy)z = xyz$ (Idempotent law)

5 **3.2.4a)**

i) True, all numbers divisible by 8 are also divisible by 4, and all numbers divisible by 4 are divisible by 2.

ii) Both inclusions are false: $2 \in A - C$ and $4 \in C - E$.

iii) False: $3 \in B - D$.

iv) True.

v) True.

vi) This is equivalent to $A \subseteq D$, which is false, as $2 \in A - D$.

b)

i) $C \cap E = E$, since $E \subseteq C$.

ii) $B \cup D = B$, since $D \subseteq B$.

iii) $A \cap B = \{n \in \mathbf{Z} | n \text{ is divisible by 2 and 3}\} = \{n \in \mathbf{Z} | n \text{ is divisible by 6}\} = D$

iv) $B \cap D = D$, since $D \subseteq B$.

v) A is the set of even integers, so $\overline{A}$ is the set of odd integers. We can write $\overline{A} = \{2n + 1 | n \in \mathbf{Z}\}$.

vi) $A \cap E = E$, since $E \subseteq A$.

6 **3.2.17b)**

$$\begin{aligned}(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B) &= ((A \cup \overline{A}) \cap B) \cup (A \cap D \cap \overline{C} \cap B) \\ &= (\mathcal{U} \cup (A \cap D \cap \overline{C})) \cap B \\ &= \mathcal{U} \cap B \\ &= B\end{aligned}$$

Here we have used distributive law, inverse law, domination law and identity law.

d)

$$\begin{aligned}\overline{A} \cup \overline{B} \cup (A \cap B \cap \overline{C}) &= (\overline{A} \cup \overline{B} \cup A) \cap (\overline{A} \cup \overline{B} \cup B) \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \\ &= (\mathcal{U} \cup \overline{B}) \cap (\overline{A} \cup \mathcal{U}) \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \\ &= \mathcal{U} \cap \mathcal{U} \cap (\overline{A} \cup \overline{B} \cup \overline{C}) \\ &= \overline{A} \cup \overline{B} \cup \overline{C}\end{aligned}$$

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$$\begin{aligned}(p \wedge (\neg s \vee q \vee \neg q)) \vee ((s \vee t \vee \neg s) \wedge \neg q) &\Leftrightarrow (p \wedge (\neg s \vee T)) \vee ((t \vee s \vee \neg s) \wedge \neg q) \\ &\Leftrightarrow (p \wedge T) \vee ((t \vee T) \wedge \neg q) \\ &\Leftrightarrow p \vee (T \wedge \neg q) \\ &\Leftrightarrow p \vee \neg q\end{aligned}$$

8 **15.1.1a)** $\overline{xy} + \overline{x} \cdot \overline{y} = \overline{1 \cdot 0} + \overline{1} \cdot \overline{0} = \overline{0} + 0 \cdot 1 = 1 + 0 = 1$

b) $w + \overline{x}y = 1 + \overline{1} \cdot 0 = 1 + 0 = 1$

15.1.2c) $\overline{x}y + xw = \overline{1}y + 1w = 0y + w = w$

The assignments of values for w and y that give the expression the value 1 are $(w, y) = (1, 0)$ and $(w, y) = (1, 1)$.

d) $\overline{x}y + w = \overline{1}y + w = 0y + w = w$, so we get the same answer as in a).

9 **15.1.10)** Suppose the variables do not all have the same value. Then one of them must be 0, which means that the right side of the equation has value 0, so $x+y+z = 0$. But this implies $x = y = z = 0$, which is a contradiction. Thus our assumption that the variables do not have the same value is wrong, which was what we wanted to prove.

15.1.11b) $x+y+\overline{\overline{x}+y+z} = x+y+\overline{\overline{x}}\cdot\overline{y}\cdot\overline{z} = x+y+x\cdot\overline{y}\cdot\overline{z} = x(1+\overline{y}\cdot\overline{z})+y = x+y$

c)

$$\begin{aligned} yz + wx + z + [wz(xy + wz)] &= (y + 1)z + wx + wzxy + wwzz \\ &= z + wx + wxzy + wz \\ &= (1 + w)z + wx(1 + zy) \\ &= z + wx \end{aligned}$$

10 Suppose $x = 0$. Then the first equation simplifies to $y = y$, and the second to $0 = 0$, so both are true. Suppose instead $x = 1$. Then the first equation becomes $1 = y$ and the second $y = 1$. We see that in both cases, the equations are equivalent, so one is true if and only if the other is.

11 **i)** $x \cdot \overline{y} + \overline{\overline{z}y}$

ii) $(0 \cdot x + y) \cdot (x + \overline{y} + z)$

iii) $(x + y) \cdot 0 + 1 \cdot x + z$

12 **i)** $xy\overline{z}yx = xxyy\overline{z} = xy\overline{z}$

ii) $xy\overline{z}y\overline{x}\ \overline{z} = (x\overline{x})y\overline{z}y\overline{z} = 0y\overline{z}y\overline{z} = 0$

13 **3.2.17a)** $A \cap (B - A) = A \cap B \cap \overline{A} = (A \cap \overline{A}) \cap B = \emptyset \cap B = \emptyset$

c) $(A - B) \cup (A \cap B) = (A \cap \overline{B}) \cup (A \cap B) = A \cap (\overline{B} \cup B) = A \cap \mathcal{U} = A$

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$$\begin{aligned} p \to (q \vee r) &\Leftrightarrow \neg p \vee (q \vee r) \\ &\Leftrightarrow \neg(p \wedge \neg q) \vee r \\ &\Leftrightarrow (p \wedge \neg q) \to r \end{aligned}$$