Project: United representation rings of finite groups

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Background: In representation theory, finite groups are studied by their linear actions on vector spaces, or equivalently, by the modules over their group rings. The representations of a finite group can be organized into a ring, the representation ring of that group. It turns out that this is a very weak invariant of the group: The two dihedral and quaternion groups of order 8 cannot be distinguished from that point of view. The purpose of this project is to define and study a refined version of the representation ring that it inspired by Bousfield's united K-theory invariants in topology.

Problem: What and how much more interesting is the united representation ring of a finite group?

Specification: (Definition) Give a precise definition of the united representation ring of a finite group. (Examples) Calculate it for small groups such as the dihedral and quaternion groups of order 8. (More ambitiously: Do the same for the Hall–Senoir groups $\Gamma_{26}a_2$ and $\Gamma_{26}a_3$ of order 64.) If time permits, try to find a way to do this on a computer. (Conjectures, theorems, proofs) Study the relation to the second Adams operation on the representation ring, formulate conjectures, and try to prove them.

Prerequisites: MA3201 (Rings and modules) and some basic group theory from MA2201 (Algebra) would be helpful. It is possible to compensate for a lack of group theory by training or at the cost of more abstraction (from group rings to semi-simple rings). MA8204 (Representation theory for finite groups) would be ideal, but the necessary parts can be explained during the training, so that it is not required.

Training: All necessary background from the representation theory of groups will be given during the supervision.

Time frame: The time frame for this project is around 100 working hours, not counting training.