Norwegian University of Science and Technology
Department of Mathematical Sciences

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Examination in TMA4115 MATEMATIKK 3
English
Monday 6th June 2011
Time: 9-13

Examination Aids (code C): Calculator (HP30S or Citizen SR-270X)
Rottman: *Collection of Mathematical Formulas*

Sensur: 27th June 2011

Full reasoning should be given for all answers. Each of the 12 parts (1, 2ab, 3ab, 4ab, 5, 6ab, 7, 8) count equal weight to the final grade.

Problem 1 Find all complex solutions of the equation

$$z^3 = \frac{1+i}{1-i}.$$

Write the solutions in the form $re^{i\theta}$, and draw the solutions in the complex plane.

Problem 2

a) Find the solution of the homogeneous equation

$$y'' - y' - 2y = 0$$

with initial conditions $y(0) = 3$ and $y'(0) = 0$.

b) Find the general solution of the equation

$$y'' - y' - 2y = 8\sin x + 3e^{2x}.$$

Problem 3 Let $y_1(x) = \frac{1}{x}$ and $y_2(x) = x^{\frac{1}{2}}$ for $x > 0$.

a) Show that y_1 and y_2 are linearly independent for $x > 0$.

b) Find an Euler–Cauchy equation which has $y = c_1 y_1 + c_2 y_2$ as its general solution.

Problem 4 Let

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 & 3 & -1 \\ 2 & 5 & -2 & 2 & 7 & 0 \\ -2 & -3 & -2 & -2 & -2 & 10 \\ 1 & 1 & 2 & 1 & 4 & 1 \end{bmatrix}.$$

a) Find a basis for the null space, Null(A) and a basis for the row space, Row(A).

b) For which values of a does the following linear system have a solution? When it has a solution, how many does it have?

$$\begin{array}{rcrcrcrcrcrcr} x_1 & + & 2x_2 & & & + & x_4 & + & 3x_5 & - & x_6 & = & a \\ 2x_1 & + & 5x_2 & - & 2x_3 & + & 2x_4 & + & 7x_5 & & & = & 1 \\ -2x_1 & - & 3x_2 & - & 2x_3 & - & 2x_4 & - & 2x_5 & + & 10x_6 & = & -1 \\ x_1 & + & x_2 & + & 2x_3 & + & x_4 & + & 4x_5 & + & x_6 & = & 0 \end{array}$$

Problem 5 Let V be the column space of the matrix

$$\begin{bmatrix} 1 & 3 & 0 & 1 \\ 2 & 1 & 5 & -3 \\ -1 & -1 & -2 & 1 \end{bmatrix}$$

and let

$$\mathbf{b} = \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}.$$

Find the nearest point in V to $\mathbf{b}$ (that is, the orthogonal projection of $\mathbf{b}$ on to V).

Problem 6

a) Let

$$A = \begin{bmatrix} 8 & 3 \\ 2 & 7 \end{bmatrix}.$$

Find the eigenvalues and corresponding eigenvectors of A.

b) There are two places in Trondheim with bicycles that can be hired for free: Gløshaugen (G) and Torget (T). The bicycles can be hired from early in the morning and must be returned to one of the places the same evening. It is found that of the bicycles hired from G, 80% are returned to G and 20% to T. Of the bicycles hired from T, 30% are returned to G and 70% to T. We assume that this pattern is constant, that all bicycles are hired out each morning, and that no bicycles are stolen.

In the long term, what proportion of the bicycles will be at Gløshaugen each morning?

Problem 7 Find a (2×2)-matrix A such that the system of differential equations $\mathbf{x}' = A\mathbf{x}$ has general solution:

$$\mathbf{x}(t) = c_1 e^{-2t} \begin{bmatrix} 3 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} -5 \\ 2 \end{bmatrix}.$$

Problem 8 Let A be an $(m \times n)$-matrix. Show that if $A\mathbf{x} = \mathbf{b}$ has solutions for all $\mathbf{b} \in \mathbb{R}^m$ then $A^T\mathbf{x} = \mathbf{0}$ has only the trivial solution.