Norwegian University of Science and Technology
Department of Mathematical Sciences

Contact during the exam:
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EXAM TMA4120 CALCULUS 4K

Monday December 14, 2009
Time: 09:00 – 13:00
Grades ready by January 18, 2010

Permitted aids (Code C): Calculator (HP 30S or Citizen SR-270X)
Rottmann: *Matematisk formelsamling*

All answers should have an explanation.
A sheet with Laplace transforms is attached at the end.

Problem 1 Find all solutions to the equation

$$e^{2z} = i.$$

Problem 2 Solve the initial value problem

$$y''(t) + 100y(t) = \delta(t-2), \qquad y(0) = y'(0) = 0$$

where $\delta(t-2)$ is Dirac's "delta function" concentrated at 2.

Problem 3 We will look at the temperature of a completely insulated bar between $x = 0$ and $x = \pi$ which satisfies the boundary value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad \text{for } 0 < x < \pi,\ t > 0$$
$$\frac{\partial u}{\partial x}(0,t) = \frac{\partial u}{\partial x}(\pi,t) = 0$$

a) Find all solutions of the form $u(x,t) = F(x)G(t)$ satisfying the boundary value problem.

b) Use the principle of superposition to find a solution which, in addition to the boundary conditions, also satisfies the initial condition

$$u(x,0) = 1 + 7\cos(3x).$$

Problem 4 First, find all the poles of the function below. Then compute the residues of the function at these poles.

$$f(z) = \frac{\sin z}{z(z+\frac{\pi}{2})(z-\frac{\pi}{2})}$$

Problem 5 Use residue calculus to compute the integral

$$\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}.$$

Problem 6 Recall that we have defined the Fourier transform of a function $f(x)$ by

$$\mathcal{F}(f) = \hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx}\, dx$$

Find the Fourier transform of the function

$$f(x) = \begin{cases} \sin(3x) & \text{for } -\pi \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$