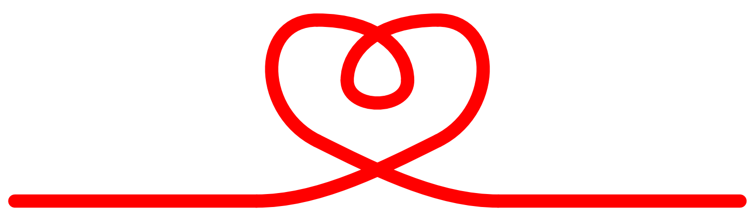
Project: An algebraic language for generic plane curves

Supervisor: Markus Szymik

Background: A plane curve is a map $\mathbb{R} \to \mathbb{R}^2, t \mapsto (x(t), y(t))$ that is continuously differentiable, with non-zero derivative, and such that $(x(t), y(t)) = (t, 0)$ for large $|t|$. A plane curve is *generic* if every point (x, y) is passed at most twice, and if so, the derivatives there are linearly independent, so that we have a *crossing*.



Two generic plane curves are equivalent if they can be deformed into each other through generic plane curves. We can classify generic plane curves up to equivalence in terms of discrete, combinatorial data. For instance, there are 42 of them with 3 crossings, and 65.004.584 with 10 crossings.

Problem: Find an algebraic description of the variety of generic plane curves in terms of simple building blocks, composition rules, and rules to eliminate redundancies.

Specification: The problem has two levels of abstraction: What kind of algebraic structure do the data describe? And which algebraic structure can be found on the set of all such data? A curve can be decomposed into points and lines, and it decomposes the plane into regions. One question is: how, conversely, these pieces can be assembled into curves. Which operations are possible? And when?

Prerequisites: There is not much more required to understand the problem. Curves can be described by drawing pictures as above. Some discrete math or general algebra might help, but it might also distract from original thinking.

Training: Because the statement of the problem is so elementary, work can start right away.

Time frame: The time frame for this project is low, about 50 working hours. Slightly more if computer experiments shall be part of it.