Exercise 1

Problems from the textbook:

6.2 Let $X_1, \ldots, X_n$ be independent random variables with densities

$$f_{X_i}(x|\theta) = \begin{cases} e^{i\theta - x} & x \geq i\theta \\ 0 & x < i\theta. \end{cases}$$

Prove that $T = \min_i(X_i/i)$ is a sufficient statistic for θ.

6.6 Let $X_1, \ldots, X_n$ be a random sample from a gamma(α, β) population. Find a two-dimensional sufficient statistic for (α, β).

6.9 For each of the following distributions let $X_1, \ldots, X_n$ be a random sample. Find a minimal sufficient statistic for θ.

(a) $f(x|\theta) = \frac{1}{\sqrt{2\pi}} e^{-(x-\theta)^2/2}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$ (normal)

(b) $f(x|\theta) = e^{-(x-\theta)}, \quad \theta < x \leq \infty, \quad -\infty < \theta < \infty$ (location exponential)

(c) $f(x|\theta) = \frac{e^{-(x-\theta)}}{(1+e^{-(x-\theta)})^2}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$ (logistic)

6.12 A natural ancillary statistic in most problems is the *sample size*. For example, let N be a random variable taking values $1, 2, \ldots$ with known probabilities $p_1, p_2, \ldots$, where $\Sigma p_i = 1$. Having observed $N = n$, perform n Bernoulli trials with success probability θ, getting X successes.

(a) Prove that the pair (X, N) is minimal sufficient and N is ancillary for θ. (Note the similarity to some of the hierarchical models discussed in Section 4.4.)

(b) Prove that the estimator X/N is unbiased for θ and has variance $\theta(1-\theta)\mathrm{E}(1/N)$.

Discuss: how/when can an ancillary statistic, whose distribution does not depend on Θ, give us information about Θ?

From previous exams:

H2020, problem 1a

H2009, problem 1a (try not to look at the solution before you make a serious effort to solve it on your own)