NTNU Workshop on PDEs

21–23 November 2018
Trondheim

1 Program

	Wednesday	Thursday	Friday
8:45-9:00	Opening		
9:00-9:40	Emmanuel Chasseigne	Erik Wahlen	Juan Manfredi
9:50-10:30	Mikko Parviainen	Didier Pilod	Erik Lindgren
10:30-11:00	Coffee	Coffee	Coffee
11:00-11:40	Tuomo Kuusi	Andre Erhardt	Grzegorz Karch
11:40-14:00	Lunch	Lunch	Lunch
14:00-14:40	Jørgen Endal	Luca Galimberti	Nils Henrik Risebro
14:40-15:20	Coffee	Coffee	Closing
15:20-16:00	Giuseppe Coclite	Peter Pang	
19:00-		Dinner	

2 Abstracts

Emmanuel Chasseigne
Title: A brief introduction to discontinuities in Hamilton-Jacobi equations and related control problems
Abstract: In 1983, the introduction of the notion of viscosity solutions by Crandall and Lions solved the main questions concerning first-order Hamilton-Jacobi equations, both for stationary and evolution cases. The theory was soon after extended for second-order elliptic and parabolic, possibly degenerate, fully nonlinear partial differential equations.

The immediate success of the notion of viscosity solutions came from both its simplicity but also universality, while it was fitting perfectly with the applications to deterministic control problems, differential games, front propagations, image analysis etc.

But despite all these positive points, there was little weakness: the theory only applies with maximal efficiency when solutions are continuous and, even more important, when the Hamiltonians in the equations are continuous. Thus, in the period 1990-2010, several attempts were made to go further in the understanding of Hamilton-Jacobi equations with discontinuities. And even more recently, several authors developed more advanced tools in the context of equations on networks.

The aim of this talk is to present in a quite non-technical way several ideas about this story, and to present some new approaches, especially in the context of control problems.

Giuseppe Coclite

Title: Nonlinear Peridynamic Models

Abstract: Some materials may naturally form discontinuities such as cracks as a result of scale effects and long range interactions. Peridynamic models such behavior introducing a new nonlocal framework for the basic equations of continuum mechanics. In this lecture we consider a nonlinear peridynamic model and discuss its well-posedness in suitable fractional Sobolev spaces.

Those results were obtained in collaboration with S. Dipierro (Milano), F. Maddalena (Bari) and E. Valdinoci (Milano).

Jørgen Endal

Title: Numerical solutions of nonlocal (and local) equations of porous medium type

Abstract: We develop a unified and easy to use framework to study fully discrete numerical methods for nonlocal (and local) equations of porous medium type. The theory includes well-posedness, stability, equicontinuity, equitightness, compactness, and convergence of the methods under minimal assumptions including assumptions that lead to very irregular solutions. In other words, the schemes we introduce are robust in the sense that they converge under very unfavorable conditions. Some numerical simulations will be presented.

This is a collaboration with Félix del Teso (BCAM) and Espen R. Jakobsen (NTNU).

Andre Erhardt

Title: Regularity results for parabolic obstacle problems

Abstract: The obstacle problem is a classic motivating example in the mathematical investigation of variational inequalities and free boundary problems. In my talk, I will present the local Hölder continuity of the spatial gradient of the solution u to the obstacle problem related to the parabolic p-Laplacian, i.e.

$$\nabla u \in C^{0;\alpha,\frac{\alpha}{2}}_{\mathrm{loc}} \quad \text{for some } \alpha \in (0,1),$$

provided the coefficients and the obstacle are regular enough. In addition, one can use the local Hölder continuity to prove the local Lipschitz continuity of the solution u, i.e.

$$u \in C^{0;1,\frac{1}{2}}_{\mathrm{loc}}.$$

Luca Galimberti

Title: Well-posedness theory for stochastically forced conservation laws on Riemannian manifolds

Abstract: We are given an n-dimensional smooth closed manifold M, endowed with a smooth Riemannian metric h. We study the Cauchy problem for a first-order scalar conservation law with stochastic forcing given by a cylindrical Wiener process W. After providing a reasonable notion of solution, we prove an existence and uniqueness-result for our Cauchy problem, by showing convergence of a suitable parabolic approximation of it. This is achieved thanks to a generalized Ito's formula for weak solutions of a wide class of stochastic partial differential equations on Riemannian manifolds.

Grzegorz Karch

Title: On a nonlinear nonlocal diffusion equation

Abstract: In the talk, I will describe an abstract framework for non-local nonlinear diffusion, by which we mean a phenomenon with properties strongly associated to diffusive processes such as the conservation of mass, the maximum principle, and the comparison principle. This framework encompasses some of the known examples of equations like the fractional porous medium equation or the equation with the fractional p-Laplacian, but it also opens up the space for new examples to be constructed and studied.

This is a joint work with Moritz Kassmann and Miłosz Krupski.

Tuomo Kuusi

Title: Numerical methods in stochastic homogenization

Abstract: I will discuss a new method for computing solutions of elliptic equations with random rapidly oscillating coefficients.

This is a joint work with S. Armstrong, A. Hannukainen and J.-C. Mourrat.

Erik Lindgren

Title: Uniqueness of extremals for Morrey's inequality

Abstract: A celebrated result in the theory of Sobolev spaces is Morrey's inequality, which establishes in particular that for a bounded domain $\Omega \subset \mathbb{R}^n$ and $p > n$, there is $c > 0$ such that

$$c\|u\|^p_{L^\infty(\Omega)} \leq \int_\Omega |Du|^p dx, \quad u \in W^{1,p}_0(\Omega).$$

Interestingly enough the equality case of this inequality has not been thoroughly investigated (unless the underlying domain is $\mathbb{R}^n$ or a ball).

I will discuss uniqueness properties of extremals of this inequality. These extremals are minimizers of the nonlinear Rayleigh quotient

$$\inf\left\{\frac{\int_\Omega |Du|^p dx}{\|u\|^p_{L^\infty(\Omega)}} : u \in W^{1,p}_0(\Omega) \setminus \{0\}\right\}.$$

In particular, I will present the result that in convex domains, extremals are determined up to a multiplicative factor. I will also explain why convexity is not necessary and why stareshapedness is not sufficient for this result to hold.

The talk is based on results obtained with Ryan Hynd.

Juan Manfredi
Title: The Strong Uniqueness Property for the p-Laplacian
Abstract: The comparison principle for viscosity solutions of many classes of non-linear equations is well-known. We consider in this talk the comparison for *viscosity solutions up to the boundary* for the Dirichlet problem for p-harmonic functions.

Given a (nice) domain $\Omega \subset \mathbb{R}^N$ and a function $g \in C(\Omega)$, a function u is a viscosity p-sub-solution if whenever a test function $\phi \in C^2(\Omega)$ touches u from above at a point x_0 we have $-\Delta_p(\phi(x_0)) \leq 0$ when $x_0 \in \Omega$ and $u(x_0) \leq g(x_0)$ when $x_0 \in \partial\Omega$. Viscosity p-super-solutions are defined analogously, and the standard comparison between p-sub-solutions and p-super-solutions holds.

To apply the results of Barles–Souganidis on convergence of monotone schemes, we need to broaden the definition of viscosity p-sub-solution (*viscosity p-sub-solution up to the boundary*) by requiring

$$\min\{-\Delta_p\phi(x_0), u(x_0) - g(x_0)\} \leq 0$$

on the boundary $x_0 \in \partial\Omega$, instead of the stronger condition $u(x_0) - g(x_0) \leq 0$. Similarly, we define *viscosity p-super-solutions up to the boundary.* Barles–Souganidis require a comparison principle for viscosity solutions up to the boundary that they call *strong uniqueness property.*

In joint work with Félix del Teso and Mikko Parviainen we establish the Strong Uniqueness Principle for p-harmonic functions, and use it so show convergence of schemes associated with many dynamic programming principles.

Peter Pang
Title: Invariant Measures of Scalar Degenerate Parabolic Equations with Additive Noise
Abstract: Degenerate parabolic equations and stochastic effects are important in modelling fluid motion and many other natural phenomena. In this short talk, building on past results on the existence and uniqueness of invariant measures to first order stochastic conservation laws, I shall discuss some recent results on existence (and, time permitting, ergodicity) of invariant measures to nonlinear anisotropic degenerate parabolic conservation laws with additive noise on a torus. I shall survey some techniques involved in proving these theorems, technical obstacles, and further interesting problems.

Mikko Parviainen
Title: Normalized equations and tug-of-war type games
Abstract: A two player zero sum game called tug-of-war with noise leads to the normalized or game theoretic p-Laplace and p-parabolic equations. In this talk,

we discuss regularity and convergence results for the normalized equations and their generalizations, as well as for the related games.

Didier Pilod
Title: Well-posedness for some dispersive perturbations of Burgers's equation
Abstract: We show that the Cauchy problem associated to a class of dispersive perturbations of Burgers' equations containing the low dispersion Benjamin–Ono equation

$$\partial_t u - D_x^{\alpha} \partial_x u + u \partial_x u = 0 \, ,$$

with $0 < \alpha \leq 1$, is locally well-posed in $H^s(\mathbb{R})$ for $s > s_\alpha := \frac{3}{2} - \frac{5\alpha}{4}$.

As a consequence, we obtain global well-posedness in the energy space $H^{\frac{\alpha}{2}}(\mathbb{R})$ as soon as $\frac{\alpha}{2} > s_\alpha$, i.e., $\alpha > \frac{6}{7}$.

This talk is based on a joint work with Luc Molinet (Université de Tours and Stéphane Vento (Université Paris 13)

Nils Henrik Risebro
Title: Conservation laws and traffic models
Abstract: We consider discrete and continuous models for car traffic on one-way roads. First we describe Braess' paradox, and how it can manifest itself in models describing networks of roads.

Then we shall explain how the standard discrete and the continuum models are related.

Finally, we shall discuss a new model for roads with many lanes. As the number of lanes increases, solutions of the many-lane models will converge to the solution of a degenerate convection-diffusion equation.

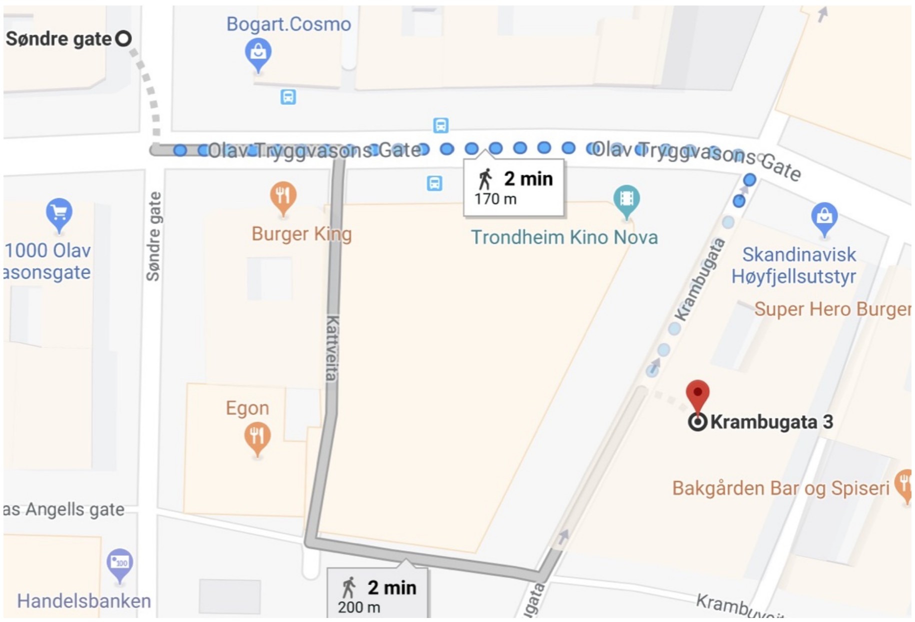
All of this pertains to joint work with Helge Holden.

Erik Wahlen
Title: Large-ampltiude solitary waves of the Whitham equation
Abstract: In the 1960's G. B. Whitham suggested a non-local version of the KdV equation as a model for water waves. Unlike the KdV equation it is not integrable, but it has certain other advantages. In particular, it has the same dispersion relation as the full water wave problem and it allows for wave breaking. The existence of a highest, cusped periodic wave was recently proved using global bifurcation theory. I will discuss the same problem for solitary waves. This presents several new challenges.

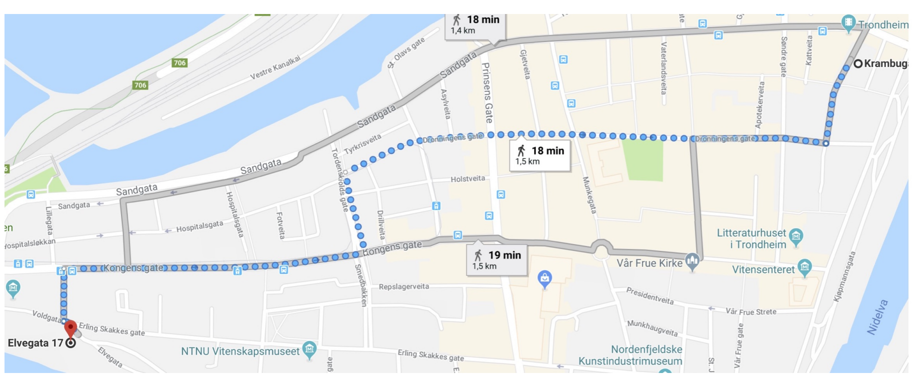
The talk is based on work in progress together with T. Truong (Lund) and M. Wheeler (Vienna).

3 Practical Information

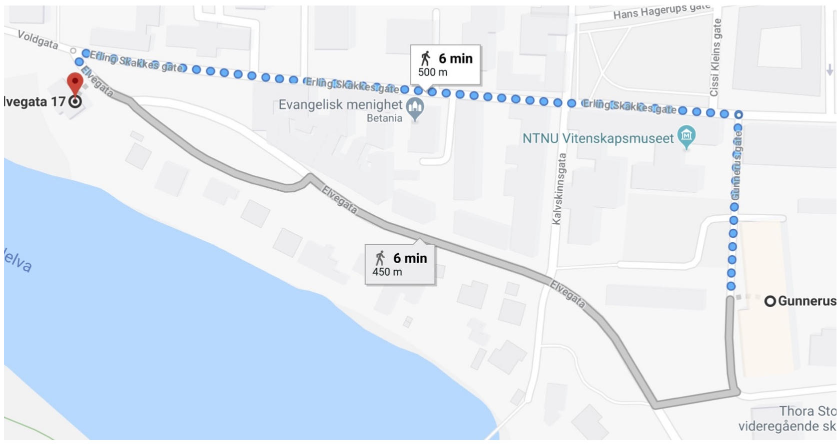
Hotel:
Comfort Hotel Trondheim, Krambugata 3, 7011 Trondheim
From the airport: Take the airport bus Flybussen til the busstopp Søndre gate. From there it is a 5 min walk to your hotel.



Talks:
will be at Royal Norwegian Society of Sciences and Letters (in Norwegian: Det Kongelige Norske Videnskabers Selskab), Elvegata 17, 7012 Trondheim.
From the hotel: It is a 20 min walk.



Lunch:
Wednesday: Will be served at the Royal Norwegian Society of Sciences and Letters.
Thursday and Friday: At Sit Kafe Kalvskinnet (Gunnerus gate 1, 7012 Trondheim). It is a 5 min walk.



Dinner:
Thursday 19:00 at Jossa Mat og Drikke, Ladeveien 9, 7066 Trondheim.
From the hotel: Either take bus number 4 in the direction Lade/Strindheim from Nova Kinosenter to Ladeveien at 18:34 or 18:47.
Alternatively it is a 30 min walk from your hotel.