Introduction

Prediction of reservoir properties from geophysical data is an inverse problem of great importance both during hydrocarbon exploration and production. Such inverse problems are a great challenge due to the measurement uncertainty and the possible non-uniqueness of the solution. Probabilistic assessment of such models are often based on Monte Carlo simulations (Mukerji et al., 2001). Seismic inversion and petro-physical inversion is often solved as a two step procedure. First one invert for the elastic attributes given seismic input, and then use the inverted elastic attributes as input in an petro-physical inversion engine to assess reservoir variables, thus often underpredicting the associated uncertainty. Rimstad et al. (2012) presented a Bayesian framework for assessment of these uncertainties.

Uncertainties are also introduced in the rock physics translation of elastic attributes to reservoir variables (Avseth et al., 2005), and the rock physics model linking rock properties and reservoir properties is generally known for conventional reservoir (Mavko et al., 2009). Unfortunately, these models may exhibit strong nonlinearities due to the presence of various lithology/fluids and saturation effects subsurface. Continuous-valued rock and fluid properties, such as porosity, are in general skewed and multimodal due to the presence of various lithology/fluids and saturation effects (Grana and Della Rossa, 2010). For example, a lithology/fluid transition from shale to gas sandstone appear as a rapid increase in porosity.

We present a Bayesian methodology for joint seismic inversion and petro-physical inversion based on a Gaussian mixture prior model for the continuous-valued variables of interest to honour these properties. The methodology is applied to pre-stack seismic and well log data from a Norwegian Sea gas discovery (Avseth et al., 2016). Assessment of the uncertainty is discussed, a problem which has received little focus in the past. Prediction of lithology/fluids is constrained by a Markov random field prior model.

Model specification

We focus on joint prediction of lithology/fluids, reservoir properties and elastic attributes subsurface given a set of seismic observations $\mathbf{d}$. For simplicity we define the model in 1D, and briefly discuss how it can be extended to 2D. The variables of interest are lithology/fluids κ, petro-physical properties $\mathbf{r}$ and elastic attributes $\mathbf{m}$, where the first variable is categorical and the latter two variables are continuous-valued. We operate in a Bayesian framework, and assessment of the posterior density:

$$p(\kappa|\mathbf{d}) = \text{const} \times \int p(\mathbf{d}|\mathbf{m})\, p(\mathbf{m}|\mathbf{r},\kappa)\, p(\mathbf{r}|\kappa)\, \mathrm{d}(\mathbf{r},\mathbf{m}) p(\kappa) \tag{1}$$

is of interest. We denote by $p(\mathbf{d}|\mathbf{m})$, $p(\mathbf{m}|\mathbf{r},\kappa)$, $p(\mathbf{r}|\kappa)$ and $p(\kappa)$, respectively, the seismic, rock-physical and petro-physical likelihood models, and the prior model. Note that the uncertainty in the lithology/fluids propagates into both the rock properties and the elastic attributes. We assume the prior model to be a first order stationary Markov chain and $p(\mathbf{r}|\kappa)$ to be a Gaussian density with expectations $\mu_{\mathbf{r}|\kappa}$ switching according to the lithology/fluids. Since both lithology/fluids, and rock properties affect the elastic attributes we define the following linear lithology/fluid dependent model for the elastic attributes:

$$[\mathbf{m}|\mathbf{r},\kappa] = \mu_{\mathbf{m}|\kappa} + \mathbf{B}_\kappa \left(\mathbf{r} - \mu_{\mathbf{r}|\kappa}\right) + \varepsilon_{\mathbf{m}|\kappa}, \tag{2}$$

where $\varepsilon_{\mathbf{m}|\kappa}$ is a centered smooth Gaussian error term. Here, $\mu_{\mathbf{m}|\kappa}$ is defined similar to $\mu_{\mathbf{r}|\kappa}$ and $\mathbf{B}_\kappa$ is a block-diagonal matrix with lithology/fluid dependent regression coefficients on the block-diagonals. The seismic likelihood is based on a convolutional model and a linearized weak contrast approximation of the Zoeppritz equation (Buland and Omre, 2003). Both $p(\mathbf{r})$ and $p(\mathbf{m})$ are Gaussian mixture densities, and it can be verified that the posterior models $p(\mathbf{r}|\mathbf{d})$ and $p(\mathbf{m}|\mathbf{d})$ are also Gaussian mixture densities.

Assessment of the normalizing constant in Eq. (1) is unfeasible because of complex spatial couplings in the likelihood models. We specify a proposal density based on a Gaussian approximation to assess Eq. (1) by Markov chain Monte Carlo (McMC) sampling . Expressions for marginal maximum posterior (MMAP) predictors are defined for the variables of interest. Extension to 2D is done by a block-Gibbs algorithm, where the prior model is a Markov random field (Rimstad et al., 2012).

Case study from the Norwegian Sea

We demonstrate the proposed methodology on a cross section from the Norwegian Sea based on near, mid and far angles-stacks. Well log data for rock physics calibration include P- and S-wave velocities, density, and various petro-physical logs (porosity and saturation). Three distinct lithology/fluids are encountered in the target zone, namely, shale, brine sandstone and gas sandstone. The continuous-valued variables of interest are $\mathbf{r} = (\phi, \mathbf{s}_w)$ and $\mathbf{m} = \left(\mathbf{i}_p, \mathbf{v}_p/\mathbf{v}_s\right)$. The gas reservoir is characterized by a porosity of 25-35 %, and we refer to Avseth et al. (2016) for further details.

Empirical rock-physics models are calibrated to upscaled well log data from the target interval for different lithology/fluid categories, see Fig. 1. Note that the rock-physics models are 2D surfaces mathematically defined for all saturations ([0,1]) and porosities ([0,1]), however, only a subset of the model surfaces are valid geophysically. For instance, the rock physics model for brine sandstone is mathematically defined for all water saturations. However, the model probabilities for brine sandstone are in practice zero for commercial gas saturations ($s_w < 0.5$). We assume a patchy saturation pattern at the seismic scale, implying close to linear relationship between saturation and impedances. If gas saturation is lower than 0.5 in sandstone (non-commercial), we will classify the rock as brine sandstone.

An average acceptance rate of 17 % is obtained in the McMC algorithm. Predictions are cross-validated at the well location, see Fig. 2 and Fig. 3. We observe that we are partly able to identify the lithology/fluid transitions, however, we loose small-scale variability in the MMAP predictor, as expected. Results for the inverted section 2D is displayed in Fig. 4. In Tab. 1 we present the 80 % coverage ratios and root mean squared prediction errors for the continuous-valued variables. The values appear to be satisfactory, however, the coverage ratio for the $\mathbf{v}_p/\mathbf{v}_s$-ratio is somewhat low.

Conclusions

The proposed methodology allows for joint probabilistic assessment of lithology/fluids, reservoir properties and elastic attributes, and is demonstrated on a cross section. Only an empirically calibrated rock physics model is considered, however, first-order Taylor expansions of known rock-physics models, for example Dvorkin's cemented sand model, is feasible. We demonstrate the methodology on seismic AVO data from a Norwegian Sea gas field, and obtain a good match with the observed lithology/fluid transitions at a selected well location.

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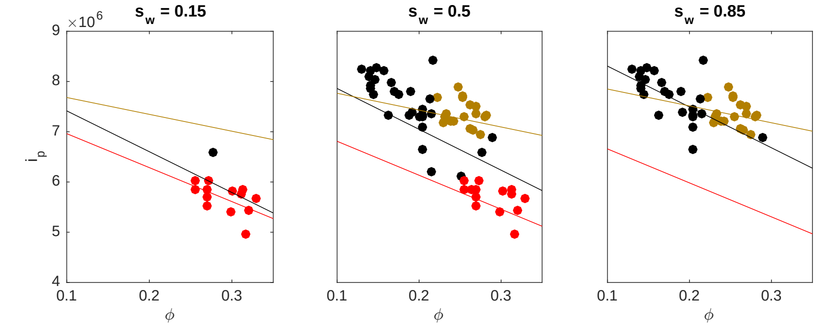

Figure 1: Upscaled calibrated rock physics models for $[\mathbf{i}_p|\kappa,\phi,\mathbf{s}_w]$ for three fixed values of water saturation. Each fitted regression lines is a function of porosity, and a fixed lithology/fluid and water saturation. Observed lithology/fluids (shale in black, brine sandstone in brown, and gas sandstone in red) at the well location are displayed if their respective water saturation value are inside respectively [0.0,0.3], [0.0,1.0] and [0.7, 1.0].

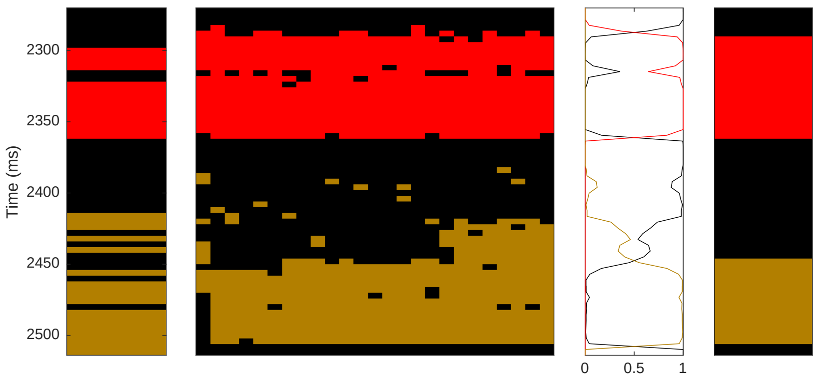

Figure 2: Posterior results lithology/fluids at the well location. Observed lithology/fluids at well location (shale in black, brine sandstone in brown, and gas sandstone in red), conditional realizations, marginal probabilities and MMAP predictor.

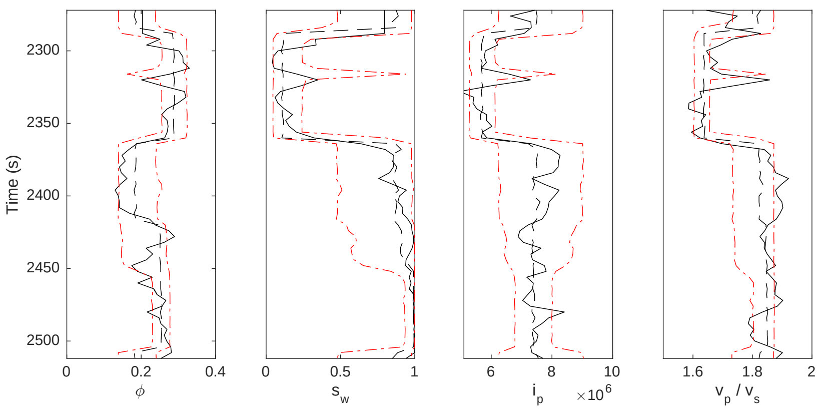

Figure 3: Posterior results for the continuous-valued variables at the well location. Observed ϕ log (solid black line), MMAP predictor (dashed black line) and 80 % prediction interval (dashed red line). Results are displayed in a similar format for $\mathbf{s}_w$, $\mathbf{i}_p$ and $\mathbf{v}_p/\mathbf{v}_s$.

Table 1: 80 % coverage ratios and root mean squared prediction errors.

	ϕ	$\mathbf{s}_w$	$\mathbf{i}_p$	$\mathbf{v}_p/\mathbf{v}_s$
Coverage ratio	0.77	0.89	0.93	0.48
RMSE	0.04	0.12	4.8×10^5	0.07

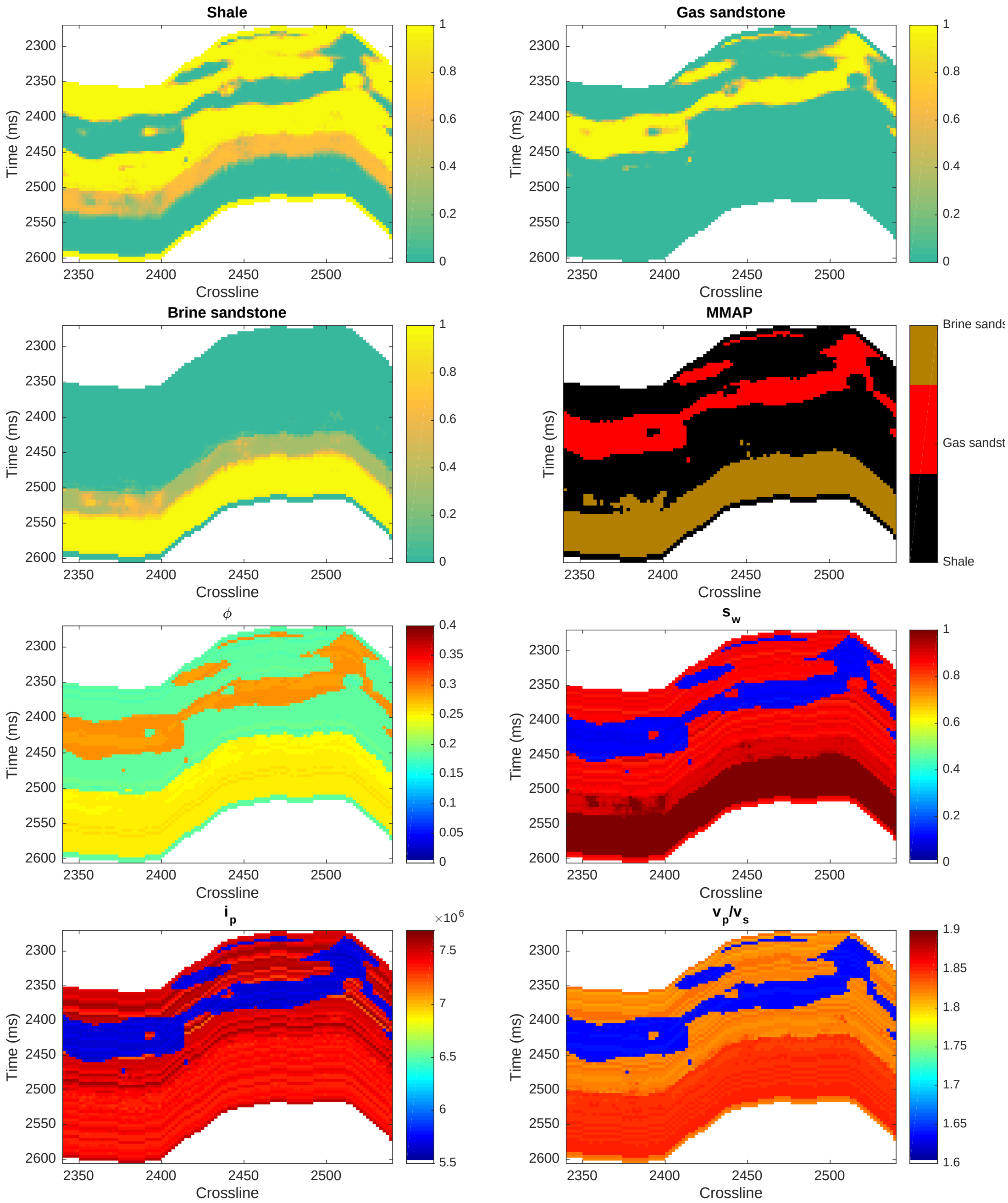

Figure 4: Posterior results for 2D cross section. Probability maps of lithology/fluids and MMAP predictor, and MMAP predictors for reservoir properties and elastic attributes.