Bayesian inversion based on Gaussian mixture prior models with applications in reservoir characterisation

Consider a profile with categorical classes subsurface penetrating a reservoir unit. Bayesian inversion methods (Tarantola, 2005) are popular in geophysics and petroleum engineering for solving complex inverse problems and predicting variables of interest.

We define a convolved hidden Markov model including three hidden layers extending the model in Larsen et al. (2006). The hidden layers are defined by the following variables: lithology/fluid classes, the rock properties and the elastic attributes, respectively. One categorical variable and two continuous-valued variables. The observations appear as a spatial convolution of the elastic attributes.

We assume a first-order Markov chain prior model for the categorical variable. The rock properties are assumed to be a Gaussian spatial variable conditional on the lithology/fluid classes. We specify a Gauss-linear rock physics model (Avseth et al., 2005) for the elastic attributes dependent on the lithology/fluid classes and the rock properties. Indeed, the marginals of the two continuous-valued variables appears as Gaussian mixture spatial variables. A set of convolved seismic reflections are observed with additive Gaussian errors.

We operate in a Bayesian inversion framework, and the objective is to predict lithology/fluid classes, rock properties and elastic attributes subsurface given seismic amplitude versus offset observations. Due to the spatial convolution and spatial coupling in the likelihood model, straightforward assessment of the posterior model is unfeasible because of the challenging normalizing constant.

A class of k-th order likelihood approximations, extending Fjeldstad and Omre (2017), is defined to construct approximate posterior models on factorial form which can be exactly assessed by recursive algorithms (Reeves and Pettit, 2004). The approximate posterior model is used as proposal density in an independent proposal Markov chain Monte Carlo Metropolis Hastings algorithm to assess the correct posterior model. The associated MMAP predictor and uncertainty statements are given for the categorical variable. Assessment of the other two unobservable layers, which are Gaussian mixture spatial random variables, is then straightforward. We use a MMAP predictor for the continuous-valued properties, where uncertainty in the categorical variable propagates into the uncertainty of the continuous-valued variables.

A 1D simulation study inspired by seismic inversion is presented to empirically evaluate the proposed class of approximation. Reasonable mixing and acceptance rates are obtained even for small k. The proposed methodology is generalized to 2D models, and a case is presented on a seismic 2D cross section based on real data including an exploration well.