Introduction

Pore pressure build-up and release is an important part of subsurface modelling because it is a result of geological processes that control the subsidence and compaction of sedimentary basin over time. It is also essential for the understanding of current day pore pressure distributions, which are important in exploration and development drilling operations. Accurate pore pressure prediction helps avoid drilling risks as it allows improved tuning of the mud weights to avoid kicks, and to reduce drilling costs by wisely choosing the casing point before entering the reservoir or some high-pressure formation, see e.g. Gholami et al. (2015).

The focus of this work is pore pressure prediction from 3D pressure modelling of basin scale. We present a new approach for online prediction of pore pressure, using the pre-drill assessment as a prior distribution, and updating the pore pressure distribution when new measurements get available while drilling. The approach provides online spatial pore pressure prediction, meaning that the pore pressure distribution ahead of the bit and at other lateral and depth locations is also updated while drilling.

Since the predictions of pore pressure are commonly applied to make decisions about the mud weight and casing points, it is critical to get realistic level of the uncertainty in the prediction (Wessling et al., 2013). Our Bayesian modelling formulation naturally allows for consistent uncertainty quantification as part of the workflow. Bayesian statistics have been applied to pore pressure prediction previously by Oughton et al. (2017), who use a Bayesian network model to connect the pore pressure variables at different depth, and to different kinds of data. Bektas et al. (2015) applied a sequential Bayesian modelling approach, which is in a similar vein to what we are doing here, but without formalising the prior and likelihood models specification from data.

Methodology

The pore pressure is denoted $\boldsymbol{p} = (p_1, \ldots, p_n)$, where $p_i = p(\boldsymbol{s}_i)$ is the pore pressure at spatial location $\boldsymbol{s}_i = (s_{i1}, s_{i2}, s_{i3})$, represented by north, east and depth coordinates. The n sites are located on a grid covering the 3D domain of interest. The pre-drill information about pore pressure consists of geological understanding of the basin environment and analysis of large-scale seismic data. Well log data are used to update the pre-drill knowledge about pore pressure. The well log data are denoted $\boldsymbol{y}_j$, where the index j refers to data collected over the well path order $j = 1, \ldots, N$, where N is the total number of data points considered. We can further clarify this by using notation $\boldsymbol{y}_j(\boldsymbol{s}_{w,j})$, indicating that the well is at spatial location $\boldsymbol{s}_{w,j}$ at step j. In our case we consider resistivity logs (r), porosity (ϕ) and acoustic logs of traveltime (Δt), so that data are $\boldsymbol{y}_j = (r_j, \phi_j, \Delta t_j)$, $j = 1, \ldots, N$.

Prior model - pre-drill assessment. We construct a multivariate prior distribution for pore pressure variables in the subsurface domain by using extensive pre-drill assessment based on 3D pressure modelling. To keep the pore pressure with values between the hydrostatic pressure ($\boldsymbol{p}_h$) and the overburden stress ($\boldsymbol{p}_{ob}$), and because statistical modelling with constraints can be difficult, we introduce the transformed pore pressure variable $\boldsymbol{x} = (x_1, \ldots, x_n)$, where $x_i(p_i) = \log\left(\frac{p_i - p_{h_i}}{p_{ob_i} - p_i}\right)$, $i = 1, \ldots, n$. The prior distribution for $\boldsymbol{x}$ is a multivariate Gaussian, i.e. the prior density is $\pi(\boldsymbol{x}) = \text{Normal}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where the mean vector is $\boldsymbol{\mu}$ and the variance-covariance matrix is $\boldsymbol{\Sigma}$.

The prior mean and variance-covariance are specified from data. In our case, this pre-drill information is given by Pressim, which is a software developed by SINTEF Petroleum Research (Borge, 2000). Important factors are then included to model stresses and pressures over geological time, up to present day. The uncertainty in pressure build-up and release is studied using a Monte Carlo approach (Lothe et al., 2004) where the inputs to the geological model vary (Lothe et al., 2018). In our setting, we only consider a few Monte Carlo realisations and we only study the pore pressure at the present time. The mean is set from pore pressure depth trends in a number of identified geological layers. The variance-covariance is set from the variability in pore pressure in these layers as well as the correlations in the lateral and depth directions which are studied from geo-statistical variograms. In future, we assume that in other dataset, we have a larger lateral variation in the measured pressures due to compartmentalisation controlled by

faults. The mean should be selected from the Monte Carlo 3D basin scale pressure modelling, and not from single pre-drill runs.

Even though we always work with the transformed variable x_i, we plot and interpret results for pore pressure p_i. The distribution for pore pressure can be obtained using for instance a first order Taylor expansion centred in μ_{x_i}. Figure 1 shows the prior mean and uncertainty. This is plotted together with the hydrostatic pressure and the overburden stress.

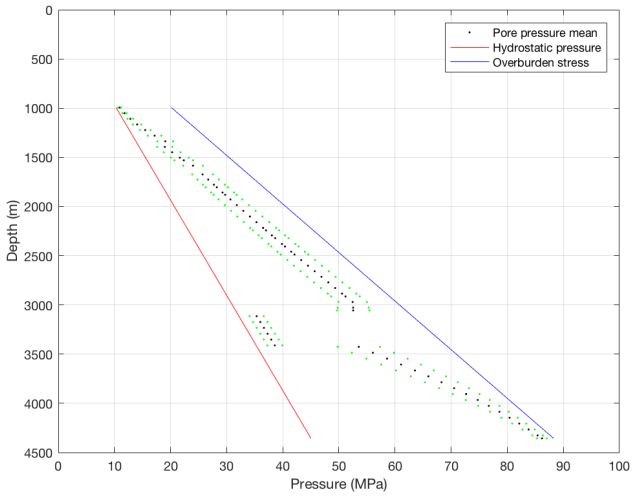

Figure 1: Prior mean with a 90% prediction interval (green)

Likelihood of well log data. The likelihood model should describe the distribution of well log responses as a function of pore pressure. There are several rock physics relations linking pore pressure to petrophysical and geophysical variables. However, they are often complicated by multi-variable interactions in the relations, and they tend to work in specific depositional environments. For instance, porosity clearly depends on pore pressure, but it also depends on the lithological composition and other attributes. So it is difficult to extract pore pressure from porosity unless we know the other variables going into the equation. The same is true for other petrophysical variables. Thus, the specification of a likelihood would be case specific.

The likelihood model is in our case specified from measurements acquired in a well in the same field as used for the prior assessment. The data consist of logs within a depth domain of the well. We focus our attention on resistivity, porosity and sonic logs. Gamma ray data and resistivity, along with the layering set in the prior specification, is used in an attempt to filter out the lithologic and depth variability. We then assume that the likelihood model is a Gaussian distribution with non-linear expected values based on the rock physics relations described in Zhang (2011), and with noise variance-covariance matrix $\boldsymbol{R}$. The likelihood model for resisitvity, porosity and transit time measurements is then $\boldsymbol{y}_j = \boldsymbol{g}_j(p_j) + \boldsymbol{\epsilon}_j$, $\boldsymbol{\epsilon}_j \sim N(0, \boldsymbol{R})$, $j = 1, \ldots, N$, where $\boldsymbol{g}_j$ is the functional relationship, for depths associated with index location j. Other models would apply in other settings. The likelihood density is denoted $\pi(\boldsymbol{y}_j|\boldsymbol{x})$, and we assume that data are conditionally independent for different j. Figure 2 shows cross plots of the well log data in grey, and the functional relationship in red. Pore pressure now varies between the normal pressure and the overburden stress at a depth of 3350m. Values of the parameters for the equations are set to obtain a better fit of the data.

Sequential assimilation of data. The Bayesian formalism, with a prior model for pore pressure variables and likelihood models for the data, is suitable for consistent assimilation of multiple data sources. In our case the goal is to perform online updating of the pore pressure, at any location, when data is acquired in the well. This means that we include well log data in a step-wise manner, online, while drilling, and the posterior after one update becomes the prior for the next step, and so on.

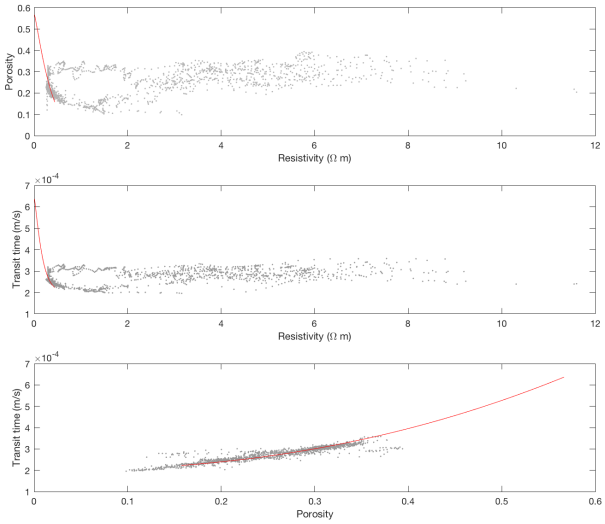

Figure 2: Cross plots of the well log data (dots) and likelihood relations as function of pore pressure (red line)

Using the transformed pore pressure variable $\boldsymbol{x} = (x_1, \ldots, x_n)$, we have online distribution

$$\begin{aligned} \pi(\boldsymbol{x}|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_j) &= \frac{\pi(\boldsymbol{x})\pi(\boldsymbol{y}_1|\boldsymbol{x})\ldots\pi(\boldsymbol{y}_j|\boldsymbol{x})}{\pi(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_j)} \\ &\propto \pi(\boldsymbol{x})\pi(\boldsymbol{y}_1|\boldsymbol{x})\ldots\pi(\boldsymbol{y}_j|\boldsymbol{x}) \propto \pi(\boldsymbol{x}|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{j-1})\pi(\boldsymbol{y}_j|\boldsymbol{x}). \end{aligned} \tag{1}$$

Here, we use the assumption that consecutive measurements along the borehole, for $j = 1, \ldots, N$, are conditionally independent, given the pore pressure variables. The distribution in equation (1) is assessed by a linearized approach, not dissimilar to the extended Kalman filter (Särkkä, 2013). For the situation with a Gaussian prior and a linearized Gaussian likelihood model, the sequential updating of data leads to the Gaussian distribution for $\pi(\boldsymbol{x}|\boldsymbol{y}_1, \ldots, \boldsymbol{y}_j)$ in equation (1).

Results

The idea here is to study a new case, where we re-play a well centred in a 3D subsurface grid, and study the effect of assimilating data. The regular grid is 10×10 in the north-east direction ((x,y) plane), where each cell is 50×50 m^2. In depth (z direction) we keep a structure similar to the prior specification, in particular we keep the same division in layers. Data are gathered along a vertical well, the data assimilation starts at 1674m and terminates at 3056m.

The sequential updating method, based on this data, is applied to $\boldsymbol{x}$ and then transformed to $\boldsymbol{p}$ for visualisation. Results are shown for pore pressure at an intermediate step (Figure 3) where data are collected up to the depth of 2913m. The left plot in Figure 3 displays the pore pressure prediction for the sites along the well path with a 90% prediction interval, while the right plots shows the updated standard deviation and mean for a lateral grid at the depth of 3056m.

Conclusions

The main contribution of our study is pore pressure prediction with a particular attention to the **Bayesian modelling** that provides consistent integration of pre-drill a priori knowledge and the well log measurements; to the **online spatial prediction**, because the prediction is updated when new well log data are available and is done ahead of the bit and at other lateral and depth locations; and to the **uncertainty** since the spatial predictions of pore pressure are represented by a mean value best prediction and a variance/covariance description.

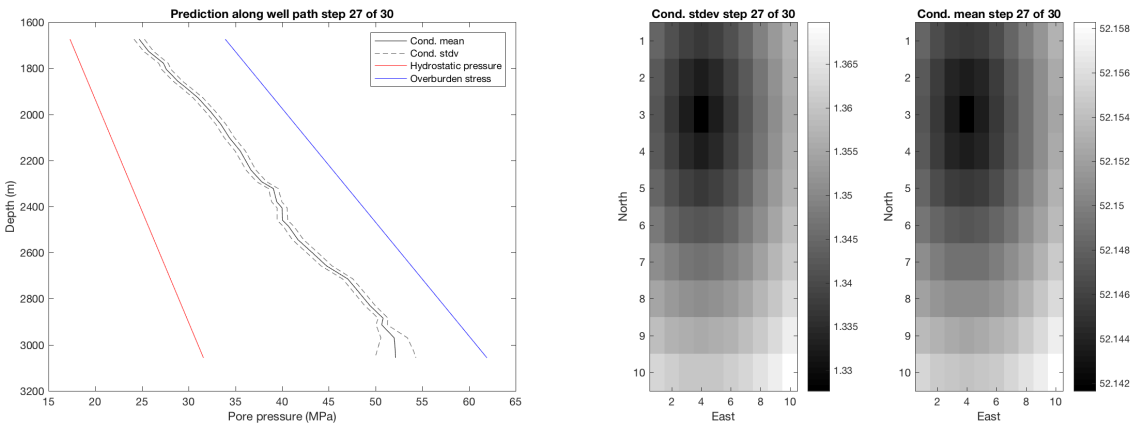

Figure 3: Intermediate step of the sequential updating

The workflow we used for our particular case is based on building a prior model from pre-drill assessment, a likelihood model from available well logs, and running sequential Bayesian updating. Although this is a fit-for-purpose routine, it is quite flexible and be adapted in various situations. For instance, it can be applied to pre-drill assessment where there are large uncertainty in depth trends, or for including other variables than pore pressure, such as more detailed information about facies classes. Also, the measurements can change to other kinds of data than what was considered here. For instance, one can evaluate the information content in look-ahead tools by re-playing a well using such data.

Acknowledgements

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