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Bayesian Generalized Gaussian inversion of seismic data.

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SUMMARY

Bayesian Gen-Gauss inversion is defined and it is demonstrated that it has great flexibility. The model is successfully used to invert seismic AVO data, and approximately 30% improvements in MSE of Bayesian Gauss inversion is obtained. The model extends easily to 3D, although the computational demands will increase considerably.

Introduction

Bayesian Gaussian inversion is frequently used in seismic AVO inversion. The Gaussian assumption in the prior model for the log-transformed elastic material properties is often difficult to justify. We define Bayesian Generalized Gaussian inversion which relies on a more general prior model.

Consider a profile through a reservoir discretized into $(1,.,t,.,T)$. The objective is to assess the log-transformed elastic material properties along the discretized profile $\boldsymbol{m} = (\log \boldsymbol{v}_p, \log \boldsymbol{v}_s, \log \boldsymbol{\rho})$. The histograms of the elastic properties from a comparable profile are displayed in Figure 1. Seismic AVO data for three angles along the profile are available , $\boldsymbol{d} = (\boldsymbol{d}^{12}, \boldsymbol{d}^{22}, \boldsymbol{d}^{31})$.

Model

The assessment of the elastic material properties given the seismic AVO data $[\boldsymbol{m}|\boldsymbol{d}]$ is made in a Bayesian inversion setting, hence the posterior model is:

$$[\boldsymbol{m}|\boldsymbol{d}] \to p(\boldsymbol{m}|\boldsymbol{d}) = [p(\boldsymbol{d})]^{-1} \times p(\boldsymbol{d}|\boldsymbol{m})p(\boldsymbol{m})$$

with $p(\boldsymbol{d}|\boldsymbol{m})$ and $p(\boldsymbol{m})$ being the likelihood and prior model respectively.

The likelihood model for the seismic AVO data given the elastic properties is defined to be Gauss-linear, see Buland and Omre (2003):

$$[\boldsymbol{d}|\boldsymbol{m}] = \boldsymbol{WADm} + \boldsymbol{\epsilon}^{d|m} \to p(\boldsymbol{d}|\boldsymbol{m}) = \phi_{3T}(\boldsymbol{d}; \boldsymbol{WADm}, \boldsymbol{\Sigma}_{d|m})$$

where $\boldsymbol{W}$ is the wavelet matrix, $\boldsymbol{A}$ is the linear Zoeppritz approximation matrix, $\boldsymbol{D}$ is a difference matrix and $\boldsymbol{\epsilon}^{d|m}$ is a colored Gaussian error term.

In Bayesian Gauss inversion the prior model is assumed to be Gaussian, see Buland and Omre (2003):

$$\boldsymbol{m} \to p(\boldsymbol{m}) = \phi_{3T}(\boldsymbol{m}; \mu_m \boldsymbol{i}_{3T}, \boldsymbol{\Sigma}_m)$$

with μ_m and $\boldsymbol{\Sigma}_m$ being the expected level and spatial covariance matrix, respectively.

This prior model choice entails that the marginal distributions of the elastic properties are Gaussian, and hence uni-modal and symmetric. In practice we often observe multi-modal marginal distributions for these properties since they depend on underlying lithologies, see Figure 1. Consequently, the prior model should ideally capture this multi-modality.

A more flexible prior model can be defined by using a selection Gaussian consept, see Arellano-Valle et al (2006). This approach is used in Rimstad and Omre (2014a, 2014b) to define a generalized Gaussian prior model which may appear with marginals that are skewed, heavitailed and/or multi-modal:

$$\begin{aligned}\boldsymbol{m} \to p(\boldsymbol{m}) &= [\Phi_{3T}(\cup_{t=1}^{3T}\Psi_\nu; 0\boldsymbol{i}_{3T}, \tau_\nu^2\boldsymbol{\Sigma}_s^\rho + (1-\tau_\nu^2)\boldsymbol{I}_{3T})]^{-1} \\ &\times \prod_{t=1}^{3T} \Phi_1(\Psi_\nu; \tau_\nu[\sigma_s]^{-1}(m_t - \mu_s), 1-\tau_\nu^2)\phi_{3T}(\boldsymbol{m}; \mu_s\boldsymbol{i}_{3T}, \sigma_s^2\boldsymbol{\Sigma}_s^\rho)\end{aligned}$$

with $\Phi_n(\boldsymbol{\Psi}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ being the probability for $\{\boldsymbol{x} \in \boldsymbol{\Psi}\}$ with $\boldsymbol{x} \to \phi_n(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$ and the set $\boldsymbol{\Psi} \in \mathcal{R}^n$.

The definition of the Gen-Gauss prior model is based on a $(6T \times 1)$ vector:

$$\begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{\nu} \end{bmatrix} \to p\left(\begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{\nu} \end{bmatrix}\right) = \phi_{3T+3T}\left(\begin{bmatrix} \boldsymbol{s} \\ \boldsymbol{\nu} \end{bmatrix}; \begin{bmatrix} \mu_s\boldsymbol{i}_{3T} \\ 0\boldsymbol{i}_{3T} \end{bmatrix}, \begin{bmatrix} \sigma_s^2\boldsymbol{\Sigma}_s^\rho & \sigma_s\tau_\nu\boldsymbol{\Sigma}_s^\rho \\ \sigma_s\tau_\nu\boldsymbol{\Sigma}_s^\rho & \tau_\nu^2\boldsymbol{\Sigma}_s^\rho + (1-\tau_\nu^2)\boldsymbol{I}_{3T} \end{bmatrix}\right)$$

where $\boldsymbol{m}$ is defined by a selection consept with respect to a selection set $\Psi_\nu \in \mathcal{R}^1$, see also Figure 2:

$$\begin{aligned}\boldsymbol{m} = [\boldsymbol{s}|\boldsymbol{\nu} \in \cup_{t=1}^{3T}\Psi_\nu] \to p(\boldsymbol{m}) &= p(\boldsymbol{s}|\boldsymbol{\nu} \in \cup_{t=1}^{3T}\Psi_\nu) \\ &= [p(\boldsymbol{\nu} \in \cup_{t=1}^{3T}\Psi_\nu)]^{-1} \times p(\boldsymbol{\nu} \in \cup_{t=1}^{3T}\Psi_\nu|\boldsymbol{s})p(\boldsymbol{s})\end{aligned}$$

The model parameters μ_s and σ_s^2 represent location and scale respectively, and $\boldsymbol{\Sigma}_{\boldsymbol{s}}^{\boldsymbol{\rho}}$ is a spatial correlation matrix, while τ_ν and $\Psi_\nu \in \mathcal{R}^1$ represents shape characteristics.

The Gen-Gauss class of prior models share several important characteristics with the Gauss class of prior models. The most important ones for us being closedness with respect to: a) marginalization, b) linearization, c) merging of independent components and d) conditioning. All these characteristics are important for making analytical assessment of the posterior model in Bayesian Gen-Gauss inversion possible, see Karimi et al (2010) and Rimstad and Omre (2014a,2014b).

In Bayesian inversion with a Gauss-linear likelihood model and a Gen-Gauss prior model, the posterior model will be Gen-Gauss with analytically assessable model parameters:

$$
\begin{aligned}
[\boldsymbol{m}|\boldsymbol{d}] = [\boldsymbol{s}|\boldsymbol{\nu} \in \cup_{t=1}^{T} \Psi_\nu, \boldsymbol{d}] \rightarrow p(\boldsymbol{m}|\boldsymbol{d}) &= [p(\boldsymbol{\nu} \in \cup_{t=1}^{T} \Psi_\nu | \boldsymbol{d})]^{-1} \\
&\times p(\boldsymbol{\nu} \in \cup_{t=1}^{T} \Psi_\nu | \boldsymbol{m}, \boldsymbol{d}) p(\boldsymbol{m}|\boldsymbol{d})
\end{aligned}
$$

Example

The characteristics of Bayesian Gen-Gauss inversion is presented in Figure 3. The spatial variable is in this example represented on a 1D grid $\boldsymbol{r} = (r_1, \ldots, r_{128})$. The likelihood model provides two exact observations $\boldsymbol{r}^o = (r_{16}, r_{112})$. We define four prior models, which define Case 1 thru 4. The marginal pdfs of the prior models are displayed in the left column, with the respective selection sets $\Psi_\nu \in \mathcal{R}^1$ presented as grey bars. Note the large variety in multi-modes, skewness and heavitailedness. We use a relatively smooth spatial correlation structure. The middle column displays conditional realizations $[\boldsymbol{r}|\boldsymbol{r}^o]$ under varying prior models. Note the mode-jumping appearance in Case 1 and 2. The right column display predictions under various prediction criteria co-displayed with kriging under comparable Gauss prior models. The conditioning on the exact $\boldsymbol{r}^o$ can be observed, and the conditional expectation predictors do not deviate much from kriging. For multi-modal prior models, expectation will often appear at low-probability values, however, and conditional mode predictors may be preferable. These mode predictors deviate considerably from kriging, which of course also is the mode predictor under Gaussianity.

Case Study

Real elastic properties from a North Sea field is used, see Figure 5 and synthetic seismic AVO data is generated, see Figure 4. Bayesian Gauss and Gen-Gauss inversion is performed, see Rimstad and Omre (2014b) for more details. The marginal pdfs of the two prior models are displayed in Figure 1. The predictions from the two inversions are co-displayed with the reference elastic property profile in Figure 5. Conditional expectation predictors are presented since they do not deviate much from mode predictors in this case with abundance of seismic data. Note that the Gen-Gauss model reproduces the steps in the reference variables better than the Gauss model. In Figure 6, conditional realizations under the two models are displayed. Note that the Gen-Gauss model reproduces the bi-modality of two of the elastic properties as introduced through the prior model. Lastly, the MSE of the predictions listed in Table 1, documents an improvement of about 30% by using Bayesian Gen-Gauss inversion over the Gauss model.

Conclusion

Bayesian Gen-Gauss inversion is defined and it is demonstrated that it has great flexibility. The model is successfully used to invert seismic AVO data, and approximately 30% improvements in MSE of Bayesian Gauss inversion is obtained. The model extends easily to 3D, although the computational demands will increase considerably.

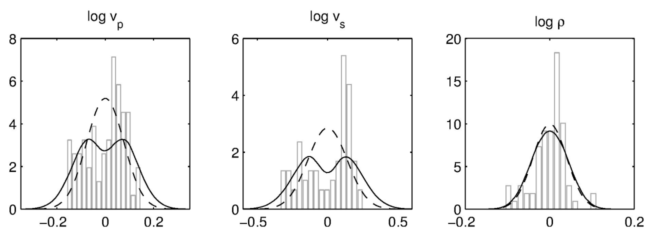

Figure 1: Histograms of log-transformed elastic material properties from comparable profile. Best fit of prior Gauss (hatch) and Gen-Gauss model (solid).

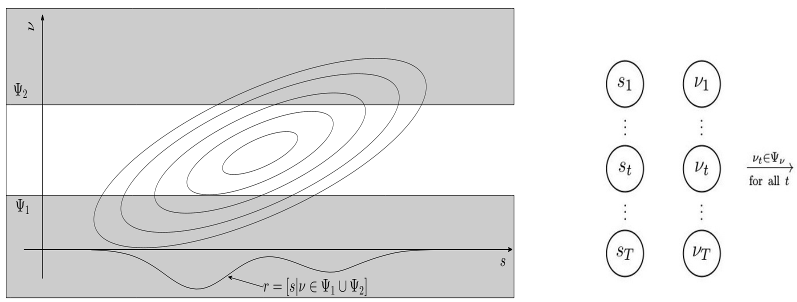

Figure 2: Construction of pdf for uni-variate Gen-Gauss r (left) and multi-variate Gen-Gauss $\boldsymbol{r}$ (right).

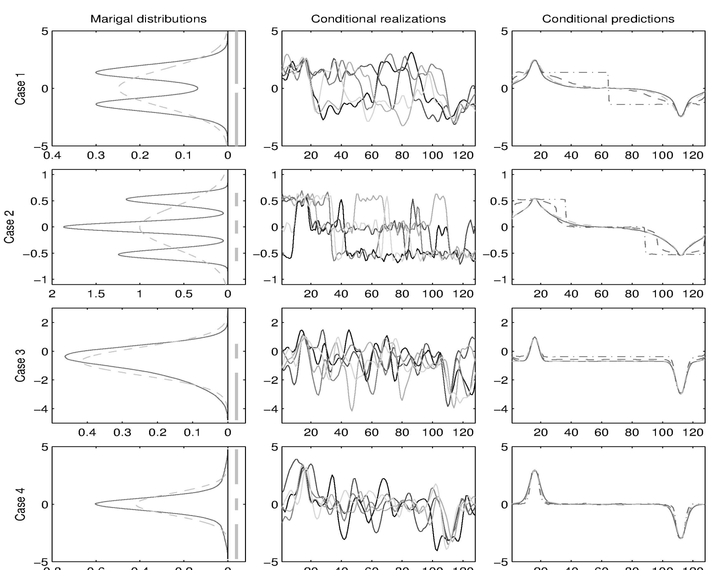

Figure 3: Examples of Gen-Gauss random fields conditioned on two exact observations. Left: Marginal pdfs (solid) and selection set (grey) with best Gauss fit (grey hatch), Middle: Set of conditional realizations, Right: Predictions - conditional expectations Gauss (solid), Gen-Gauss (hatch) and maximum posterior modes Gauss (solid), Gen-Gauss (hatch-dot).

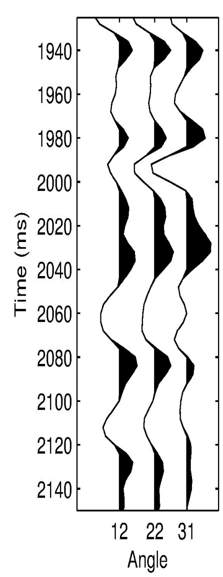

Figure 4: Case study - reference seismic AVO data

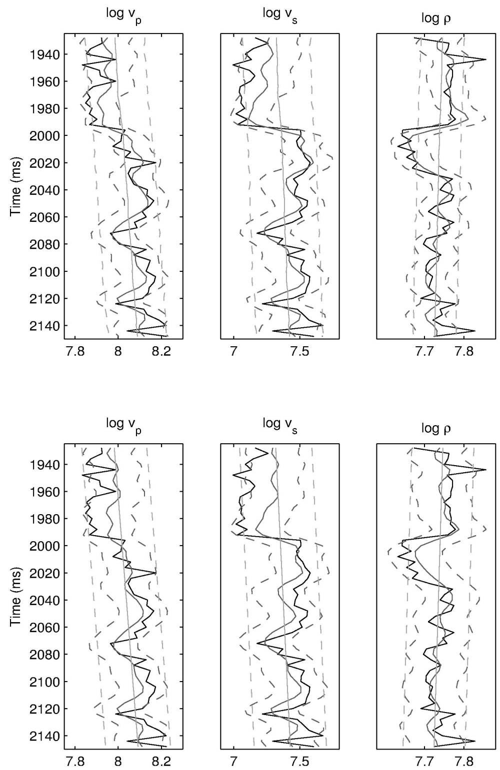

Figure 5: Conditional expectation prediction (grey solid) from seismic inversion with 90% prediction intervals (grey hatch). Reference profile (solid). Top: Gen-Gauss model, Bottom: Gauss model.

	MSE 10^{-3}	
Variable	Gen-Gauss	Gauss
$\log v_p$	3.4	5.0
$\log v_s$	11.2	19.1
$\log \rho$	0.9	1.1

Table 1: Mean Square Error for predictions versus reference profile.

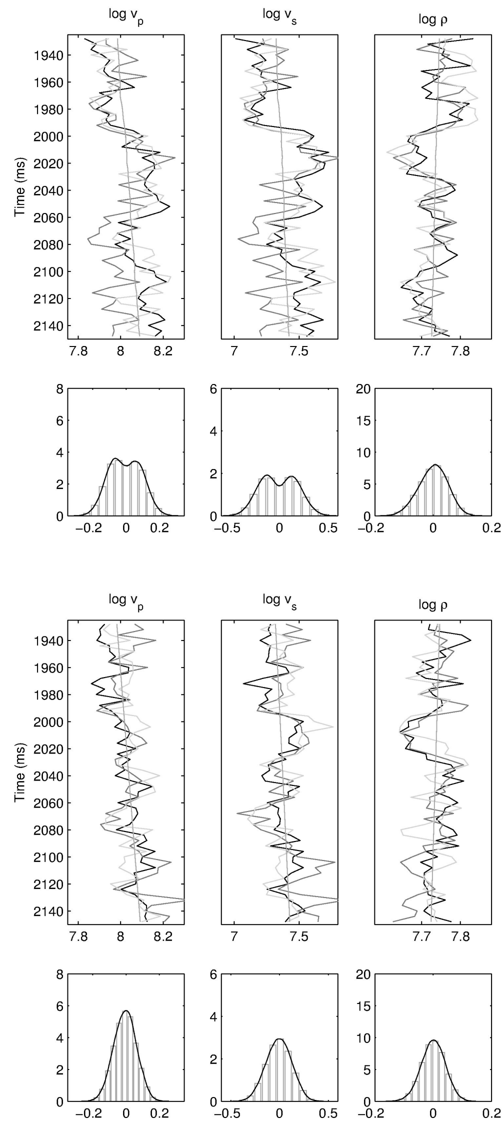

Figure 6: Conditional realizations from seismic inversion with smoothed empirical histograms. Top: Gen-Gauss model, Bottom: Gauss model

References

Arellano-Valle, R., Branco, M. and Genton, M. [2006] A unified view on skewed distributions arising from selections. *Canadian Journal of Statistics*, **34**(4), 581–601.

Buland, A. and Omre, H. [2003] Bayesian linearized avo inversion. *Geophysics*, **68**, 185–198.

Karimi, O., Omre, H. and Mohammadzadeh, M. [2010] Bayesian closed-skew gaussian inversion of seismic avo data for elastic material properties. *Geophysics*, **75**(1), R1–R11.

Rimstad, K. and Omre, H. [2014a] Generalized gaussian random fields using hidden selections. *arXiv 1402.1144 [stat.ME]*, 1–26.

Rimstad, K. and Omre, H. [2014b] Skew-gaussian random fields. *Spatial Statistics*, **10**, 43–62.