The value of sequential prospect selection

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Abstract

We present new instruments for solving the "best site selection" problem in a sequential setting, and we propose a comparison with classical non-sequential methods based on the Value of Information. The main motivation is selection of drilling sites in an oil and gas field, when there are several correlated prospects. We discuss the properties and the methods required for designing our optimal sequential program, and we present results on two case studies. Spatial statistics models are used to encode the geological spatial dependence among the sites.

1 Introduction

A classical problem in decision making theory [Bratvold and Begg, 2008] is how to optimally select a (set of) location among a set of sites that are spatially correlated, in order to optimize a certain utility function, under a number of costraints. The problem is interesting both from a conceptual and from a practical point of view. The first aspect involves the evaluation of all the implication of our choice. These are typically of two kinds: first, we have to consider how valuable is the location in itself, i.e. what is called the *intrinsic value* of the site chosen; second, we have to consider the value in terms of information provided to the whole domain of interest, i.e. how the site chosen can reduce our uncertainty about the revenues that we can collect in the future. From a practical point of view, two are the crucial aspects: the choice of the utility function, and the choice of the spatial model responsible for the correlation in the domain. The first choice entails for the decision maker a clear view and a deep knowledge of the underlying physical phenomenon. The second aspect is more general, and has raised increasing interest in the last years with the consolidation of spatial statistics techniques.

The problem of encoding spatial dependency among a set of geologically appealing locations has been tackled in different ways for different purposes. Spatial autocorrelation based models can be formulated in order to describe the correlation of an entire dataset. In order to make inference and prediction about unknown sites, data driven models such as kriging based techniques are preferred. In this case, the spatial relation of the underlying field is expressed through a function called variogram [Cressie, 1993], and a large literature is exploring how to make these instruments as flexible as possible, to keep into account heterogeneity or other spatial phenomena. Often the decision maker has already decided to organize the space in order to collect information over a pre-determined space, such as a grid: in this case lattice based models, such as Markov Random Fields (MRF), are the most indicated models to describe the spatial correlation among the data. Recent uses of such models in the oil exploration field can be found in [Bhattacharjya et al., 2010]. Along the same line of gridded based models, [Van Wees et al., 2008] first proposed a model using Bayesian Networks (BN) in stead of MRF.

In their approach the distance among the prospects was considered to be the source of spatial correlation, following a classical argument. [Martinelli et al, 2011] developed a more flexible Bayesian Network model that is able to incorporate information coming from multiple sources: from expert opinions to physical constraints. The main idea is then to exploit the information coming from experts in local geology and translate it into a correlation model for describing the hydrocarbon migration paths within the area. The BN model can be expanded to more elements present in the petroleum system, such as the reservoir, trap, and source. One such BN modeling framework is presented in [Stien et al., IAMG 2011].

The probability structure induced by a BN enjoys the Markov property, i.e. each node is conditionally independent of its non-descendants given its parents. Then the joint probability distribution for the distinction of interest, being {dry}, {gas} or {oil} at every node, can easily be factorized as:

$$p(x) = \prod_{i=1}^{N} p(x_i | x_{pa(i)}),$$

where N is the total number of nodes and $\{pa(i)\}$is the set of parents of node i. This property makes the network particularly suitable for large computations. For instance, one can exploit efficient software packages such as the Bayesian Network Toolbox building on the factorization Junction Tree Algorithm, first proposed in [Lauritzen and Spiegelhalter, 1988].

The idea is to update the probability distribution on the network after collecting information in a restricted amount of sites. The information that we are going to collect is assumed to be a *perfect information,* i.e. a precise result (in the simplest case, a *wet vs oil* outcome) of a single or a series of exploration wells placed in the considered sites. We call this information *evidence.* By repeatevely evaluating the impact of different evidences on the network our ultimate goal is to rank the sites and to provide the decision maker with a tool that helps in designing an optimal exploration strategy.

2 Best site selection: static and dynamic techniques

In this section we describe how to exploit the spatial BN models described above to design an optimal drilling strategy. The problem has been tackled in multiple ways, see [Wang and Kokolis, 2000], [Smith and Thompsons, 2008]. One of the most effective examples is offered in [Bickel and Smith, 2006] and [Bickel et al., 2008], where a dynamic programming model is developed to find the optimal drilling strategy. In [Martinelli et al., 2011], we propose the Value of Information (VoI) as a criterion for selecting the best site or pair of sites. The use of VoI in this context comes from a series of papers , see e.g. [Cunningham and Begg, 2008], [Eidsvik et al., 2008] and [Bhattacharjya et al., 2010], where this VoI index is a natural choice for deciding whether to gather or not gather more nformation (seismic, electromagnetic,...). In [Martinelli et al., 2011] the idea is to study the impact of partial perfect information (VoPI), i.e. the result of

drilling an exploration well. The sites which give higher VoPI are the most valuable to drill. The approach was defined to find the best set of exploratory wells, imposing no ordering. Thus, by a fixed (joint) selection of sites the outcome of one exploratory well did not affect the decision about the second drilling location.
We acknowledge that all this criterion can be improved by introducing a sequential approach [Miller, 1975], following the dynamic programming line discussed above. We now compare these approaches.

It is important to point out that by Perfect Information, in this context, we just mean a piece of information that is not noisy, and not a kind of information that allows us to perform the right decision with probability 1. In a way, once we gathered it, the information is perfect just for the site tested, and it is still vague for what concerns all the other sites. Nonetheless, we have chosen to use the word *perfect,* in order to distinguish it from an *imperfect* information, i.e. a piece of information that is just correlated with the exact outcome of the well, like a seismic survey. In this case we would add another level in our BN, in order to keep into account the reliability of the information collected.
It is also important to remark that BN are just one spatial model that we have chosen, but that the same procedure is appliable to different models, given that a joint probability distribution to rely on does exist.

2.1 Preliminary notation

In a simple setting with N nodes, each one with revenue R_i and cost C_i, we can formulate the standard Value of Information problem in the following terms: the site i has a positive VoI if and only the expected value that the decision maker can collect over the whole field after observing evidence on site i, is higher than the prior value, i.e. the expected value over the whole field before any evidence [Raiffa, 1968]. In mathematical terms, we have:

$$VoPI(i)=\sum_{j=1}^{N}(\,max\{E_i(R_j)-C_j,0\})-\sum_{j=1}^{N}(\,max\{E(R_j)-C_j,0\})$$

where E_i is the expectation under the measure induced by an observation in site i, while E is the standard prior expectation.

We now extend the notion by introducing the same utility function used in [Martinelli et al., 2011]. Let us consider a BN with N segments, k prospects and $n_j, j\in\{1,\dots k\}$ segments per prospect. In this setting the segments are the nodes where we look forward to drill, and the prospects are just macro-areas; the division is necessary since segments belonging to a same prospect share a number of infrastructural costs. We have a common infrastructure cost at the prospect level (DFC, drilling fixed cost), another common exploration cost at the prospect level (EFC, exploration fixed cost), and a smaller cost related to the single well exploration at the segment level (WFC, well fixed cost). Further, we have estimated volumes and revenues for all the segments in the area, and estimated development costs that we consider to be proportional to these volumes. The difference between these quantities yields the so-called Partial Revenues for Oil (PRO) and the Partial Revenues for Gas (PRG).

2.2 Value of Perfect Information

The exploration strategy under investigation can be effectively resumed in mathematical terms with a utility function. In the utility function we decide to develop or not develop a prospect, considering the potential revenues associated with the segments within that prospect, and the associated costs of prospect development and segment development. Next, given a positive decision at the prospect level, we decide which segments to develop (if any) depending on the local expected revenues.

The procedure can be summarized in the computation of the expected revenues (ER) per single segment i:

$$ER(i) = \sum_{evidence\, j} [\sum_{prospect\, k} max\{(Rev_{k|v_i=j} - DFC), 0\}]\, P(v_i = j),$$

where the revenues for prospect k, given evidence $v_i = j$, are:

$$Rev_{k|v_i=j} = \sum_{segment\, l \in k} [PRO_l \cdot P(v_l = oil \,|\, v_i = j) + PRG_l \cdot P(v_l = gas \,|\, v_i = j)]$$

According to this formula, the best site is chosen maximizing the VoPI :

$$VoPI(i) = ER(i) - \sum_{prospect\, k} max\{(Rev_k - DFC), 0\}$$

where the prior revenues for prospect k are:

$$Rev_k = \sum_{segment\, l \in k} [PRO_l \cdot P(v_l = oil) + PRG_l \cdot P(v_l = gas)]$$

This procedure can be easily generalized for couples or triplets of segments, computing for example $ER(i,j,k)$ and then $VoPI(i,j,k)$, by simultaneously conditioning on the evidence at segment i, j and k given by $\{v_i = x_i, v_j = x_j, v_k = x_k\}$.
Finally, we compare the $VoPI$ with the exploration costs for the prospects under consideration. For a prospect k, we have to take into account the following costs:

$$Cost_k = EFC_k + \sum_{segment\, l \,\in\, k} I_l \cdot WFC$$

where the indicator function I_l is 1 if $VoPI(l) > 0$.

2.3 Sequential Value of Perfect Information

In a sequential horizon [Miller, 1975] the first site is chosen in the same way, i.e. maximizing the single site $VoPI$. The second best site is then chosen according to the outcome of the outcome of the first exploration well in the following way:

$$ER(j|v_i=x_i)=\sum_{evidence\, x_j}[\sum_{prospect\, k} max\{(Rev_{k|v_j=x_j,v_i=x_i}-DFC),0\}]\,P(v_j=x_j|v_i=x_i)$$

We have

$$VoPI(j|v_i=x_i)=ER(j|v_i=x_i)-\sum_{prospect\, k} max\{(Rev_k-DFC),0\}$$

By comparing the two approaches, we see that the best two sites are given by the following maximations:

$$VoPI(s_1,s_2)_{static}=max_{i,j}\{VoPI(i,j)\}$$

$$VoPI(s_1,s_2)_{sequential}=max_i\{VoPI(i)\}+\sum_{k=1}^{3} max_j\{VoPI(j|v_{(1)}=x_k)\}P(v_{(1)}=x_k)$$

where $v_{(1)}=max_i\{VoPI(i)\}$, i.e. the $VoPI$ of the first best site.

In the operational research literature [Russell and Holsenback, 1997], the presented technique is called a myopic heuristic, and it indicates a sub-optimal optimization approach. It is often used when the combinatorial dimension of the problem does not allow an exact optimization search.

2.4 Optimal Value of Perfect Information

The sequential approach previously introduced in 2.3 has still some limits since we would like to exploit the information concerning the second best site when choosing the first site. More generally, a greedy forward search for sites does not take into account the learning between the sites. We want to be able to include the effect of $VoPI(j|i)$ in the computation of $VoPI(i)$. In this way, we get the first best two sites in the following way:

$$VoPI(s_1,s_2)_{optimal}=max_i\left\{VoPI(i)+\sum_{k=1}^{3} max_j[VoPI(j|v_i=x_k)]P(v_i=x_k)\right\}$$

We have chosen the word optimal because this approach represents the optimal procedure for computing the best choice of the first two segments. The procedure can be easily generalized to the computation of the first N segments. In this case the VoPI approach coincides exactly with the dynamic programming approach for finding the optimal drilling sequence [Bickel and Smith, 2006]. In the simple scenario presented here, when there is no explicit temporal component, we disregard the discounting factor. The computation of a full best strategy of depth N involves the evaluation of 3^N scenarios in our situation with three outcomes per node. In large case scenarios such evaluation becomes immediately unfeasible, while it is still feasible to try joint pairs of sites or a myopic strategy.

3 Results

Figure 1 shows the real case study from the North Sea [Martinelli et al., 2011], while Figure 2 shows a simple sinthetic example used for test purposes. Both networks refer just to the source part, but the approach used can be generalized to larger networks, with major dependencies.

The networks used contain three different kinds of nodes, the kitchens (K), the prospects (P) and the segments (S). The former are areas known to have undergone the right pressure and temperature conditions needed to generate hydrocarbons. The latter are places where geological conformation and seismic data have defined as promising places for exploratory wells. The prospects nodes encode the desired correlation in the model. The critical part of a BN is the definition of the conditional probability distributions encoded by arrows in the graph connecting K, P and S nodes. In this case we have a network with three possible outcomes per node: {dry}, {gas} or {oil}. The probability model exploits physical constraints and geological expert opinions.

In the case shown in Figure 1 the conditional independence assumptions imposed by the Markov structure means that any segment is independent of all the other segments given its parent prospect. For example, if the state of prospect {P4} is known, no other information can modify the probability model for segments {4A} and {4B}, since they have {P4} as a single parent.Marginally, if {P4} is not known, {4A} and {4B} is dependent of all other nodes. Say, if evidence is collected at {6C} alone, this would affect the probability of oil at {4A} and {4B}.

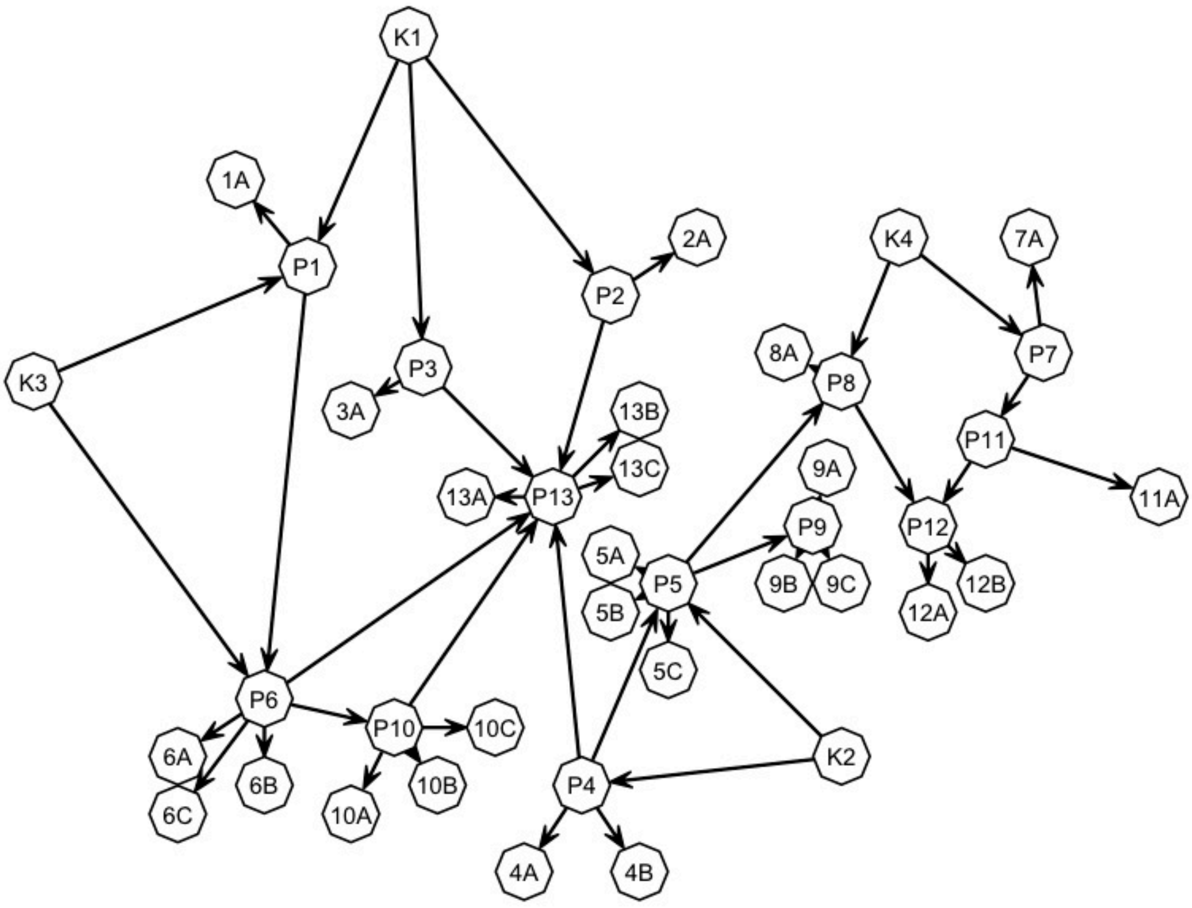

Figure 1: Bayesian Network for kitchen, prospect and segment nodes. There are 4 kitchens (K), 13 prospects (P), and 25 segments. From [Martinelli et al., 2011].

3.1 First Case Study

We first test the different approaches on a simple network with only 6 segment nodes (Figure 2). The set of parameters is shown in Table 1. It is important to note that the Intrinsic Values, i.e. the the expected value given success times the marginal probability of success (in our case oil and gas), minus the expected value given failure times the marginal probability of dry, does not include the prospect costs, that is shared among different segments. For this reason they are all positive in Table 1.

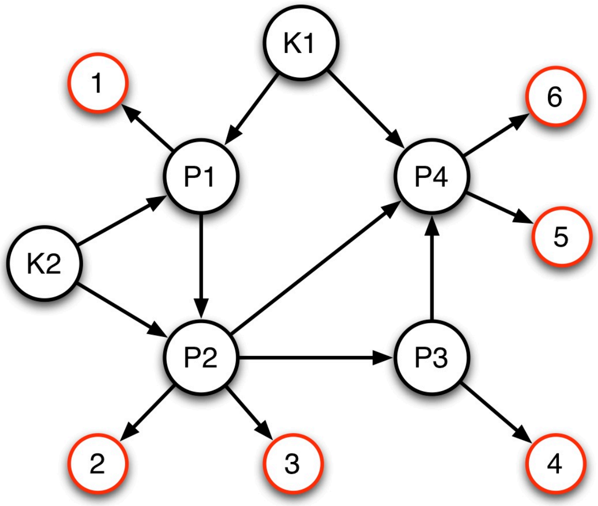

Figure 2: Bayesian Network, 1st case study.

Segment	1	2	3	4	5	6
Oil Volume (Million m^3)	6	3	5	2	7	4
Gas Volume (Billion m^3)	4	6	7	3	6	2
Marginal HC prob.	0.80	0.90	0.20	0.69	0.85	0.66
Int. Values (Billion USD)	1.37	1.51	0.45	0.63	1.74	0.55

Oil rev.	75 USD per barrel
Gas rev.	0.3 USD per m^3
WFC	20 Million USD per segment
EFC	100 Million USD per prospect
DFC	500 Million USD per prospect

Table 1: Parameters used in the 1st case study.

The results for the joint $VoPI$ calculation are presented in Table 2.

$VoPI(i)$	1	2	3	4	5	6
	99.99	198.90	104.39	164.73	66.85	0.00

$VoPI(i,j)$	1	2	3	4	5	6
1	-	298.90	204.39	273.12	171.58	101.14
2	298.90	-	318.10	329.54	264.90	212.09
3	204.39	318.10	-	260.98	198.47	126.23
4	273.12	329.54	260.98	-	242.03	187.93
5	171.58	264.90	198.47	242.03	-	83.38
6	101.14	212.09	126.23	187.93	83.38	-

Table 2: VoPI(i) (top) and VoPI(i,j) (bottom).

The static computation yields a maximum $VoPI$ of 198.90 Million USD for the first site, reached in segment {2}. The $VoPI = 0$ at segment {6}, meaning that gathering evidence at this segment is not able to make us change decision whether to drill or not drill any other site. In the bottom part of Table 2 we present the VoPI for pairs of segments. These numbers determine the best couple of sites according to the *static approach*, that in this case is represented by segments {2,4}, with a corresponding value of 329.54 Million USD. Note that the upper right and lower left part of the matrix are symmetric because the joint strategy disregards the order of the exploration wells.

When we compute the best two sites according to the sequential strategy, on the other hand, we find out that the best second choice previously indicated, segment {4}, is valid only when the segment number {2} is found *oil*, while when segment {2} is found *dry* the best choice would be drilling segment number {1}, and when segment {2} is found *gas* the best choice would be drilling segment number {3}. The resulting $VoPI$, computed according to the *sequential* approach, yields a value of 357.12 Million USD, with a substantial increase with respect to the static case in Table 2. A sketch of the best two-wells strategy is depicted in Figure 3.

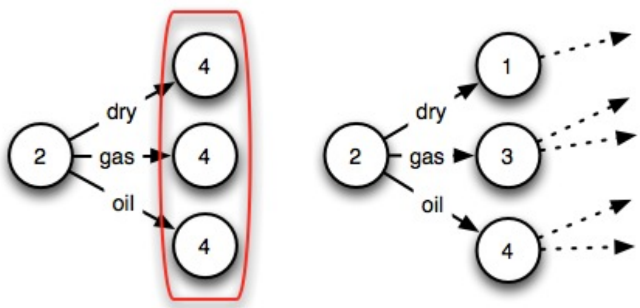

Figure 3: Best static (left) and sequential (right) strategies.

In this case the pairwise *optimal sequential approach* does not improve the sequential result since the first site is the same. The VoPI results for the 6 sites, given as a function of the first wells, are in Table 3. It is interesting to note that already in this simple case the 2-steps optimal value for segment {3} ranks second, while it ranked third in the 1-step horizon (Table 2, top). This means that site {3} alone does not provide a great information, while in combination with other segments it reveals as useful in terms of VoPI as segment {2}.

$VoPI(i,j)_{opt}$	1	2	3	4	5	6
	298.90	357.12	342.99	331.19	265.52	212.09

Table 3: VoPI(i,j)$_{opt}$ as function of i.

3.2. Second case study

We now go on to study the original motivation from a field in the North Sea (Figure 1). In Table 4 we present the set of parameters used in the case study. Moreover, we have followed what was done in [Martinelli et al., 2011], i.e. $DFC = 2\, Million\, USD$, for the drilling fixed cost.

Node	Type	Gas Vol.	Oil Vol.	Marg. Dry	Marg. Gas	Marg. Oil
1A	Segment	9.7	5.9	0.20	0.52	0.28
2A	Segment	0.4	0.5	0.40	0.21	0.39
3A	Segment	3.5	2.3	0.60	0.26	0.14
4A	Segment	0.6	0.8	0.28	0.57	0.15
4B	Segment	1.9	2.8	0.20	0.29	0.51
5A	Segment	0.5	2.4	0.21	0.04	0.75
5B	Segment	1.3	6.9	0.34	0.03	0.63
5C	Segment	2.4	14.7	0.52	0.00	0.48
6A	Segment	8.8	3.0	0.10	0.72	0.18
6B	Segment	8.8	3.0	0.20	0.64	0.16
6C	Segment	1.6	9.7	0.80	0.00	0.20
7A	Segment	0.4	1.7	0.19	0.00	0.81
8A	Segment	0.1	0.1	0.36	0.32	0.32
9A	Segment	0.6	0.2	0.10	0.45	0.45
9B	Segment	1.8	1.3	0.10	0.45	0.45
9C	Segment	0.1	1.3	0.10	0.00	0.90
10A	Segment	0.8	6.7	0.61	0.22	0.17
10B	Segment	2.2	19.0	0.30	0.00	0.70
10C	Segment	0.8	7.1	0.37	0.00	0.63
11A	Segment	3.1	1.4	0.18	0.41	0.41
12A	Segment	4.4	7.3	0.50	0.25	0.25
12B	Segment	10.3	4.8	0.41	0.47	0.12
13A	Segment	0.5	2.7	0.10	0.00	0.90
13B	Segment	8.9	3.4	0.10	0.72	0.18
13C	Segment	7.2	2.7	0.30	0.56	0.14

Table 4: Input data for the second case study: in the first two columns, name and type of nodes. Next, estimated gas (billion m3) and oil (million m3) volumes. In the last three columns, marginal probabilities of dry, gas, and oil, for all the 25 segments.

Results about single segment $VoPI$ can be found in Figure 4. As we can see, both the segments {12A} and {12B} have a large $VoPI$, respectively 745 and 819 Million USD. It is also clear that, being the two segments close to each other, neither a static two segments approach nor a sequential one would privilege segment {12A} as second best choice, and since their information largely overlaps it is clear that their coupled $VoPI$ is lower than their sum. That is exactly what we see from the results: the best myopic sequential decision suggests to drill as second site the one with higher Intrinsic Value {10B} (see Table 4), no matter the outcome of the first segment {12B}. In this case the sequential $VoPI = 1359\, Million\, USD$. In the same way, as we can see from Figure 5, the best static pair indicates segments {12B} and {10B}, with the same corresponding $VoPI = 1359\, Million\, USD$; it is interesting to note that, as said before, the joint $VoPI(\{12A\},\{12B\}) = 819\, Million\, USD$, i.e. it coincides with the larger of the two original values, with the second segment not carrying any new value, as expected. The Figure includes all the joint static $VoPI$ for the $25 * 24 / 2 = 300$ combinations of segments.

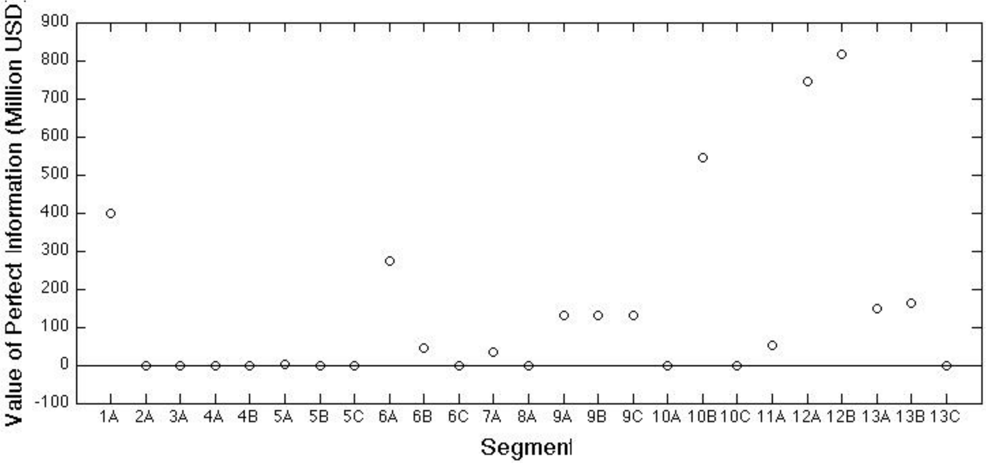

Figure 4: Single Segment VoPI, 2nd case study.

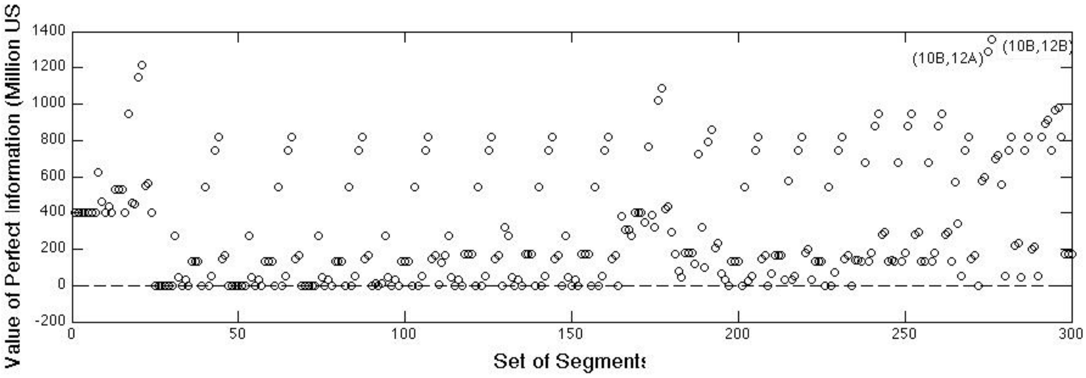

Figure 5: Two-Segments VoPI, 2nd case study.

When evaluating the optimal two-stage $VoPI$ (Figure 6) we find that, as expected, the optimal value is reached both on segment {10B} and {12B}, since, as pointed out before, there is no penalty for later exploration. It is interesting to note that when computing the optimal two segments $VoPI$, all the sites reach at least the maximum single site $VoPI$. This means that the impact of the best site can be totally additive, as it is for the large part of the segments. Otherwise, if the two sites under consideration are poorly correlated among each others, the impact of the best site can be partially or totally confounded by the mutual interaction, as already observed for the two neighbour segments {12A} and {12B}.

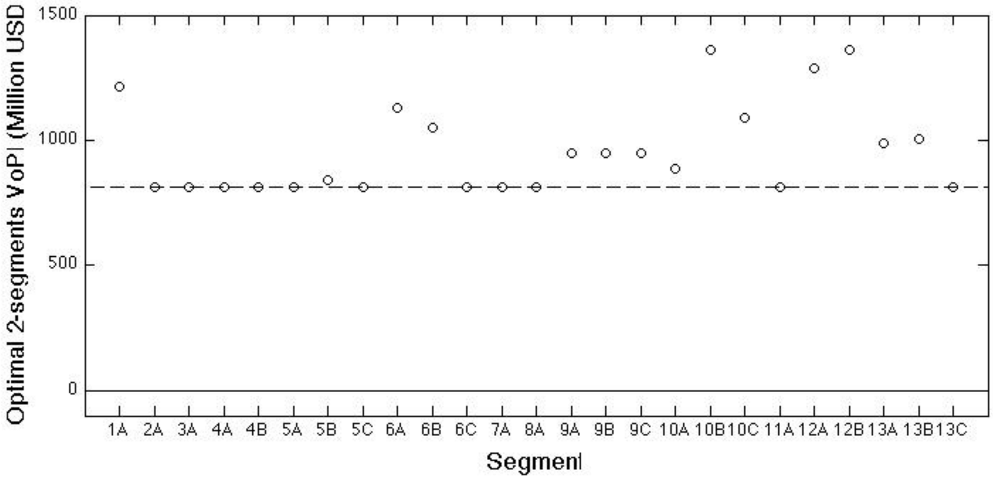

Figure 6: Optimal Two-Segments VoPI as function of the first best site, 2nd case study.

Conclusions

The main focus of this work has been to relate the Value of Information theory with sequential-based site selection approaches. With problems of higher complexity and a growing computational power available, it is of particular importance to acknowledge that a consistent sequential approach cannot be disregarded. As pointed out in [Samson et al., 1989], the Value of Information is non-additive and non-monotonic with respect to a parameters change, therefore we believe that it is interesting to show the behavior of such index in a real case study, and to deepen the consequences that it has in an early exploration stage.

This work has a natural extension in a complete Value of Information approach, with imperfect data such those ones presented in [Bhattacharjya et al., 2010]. In this case the decision does not concern the possible exploration strategy campaign, but the possible acquisition of different kinds of data, whose appraisal depends on the value added at the moment of the choice.

Another natural extension, has received much attention from us at the moment, includes adding a temporal horizon in the exploration, and penalizing the later drilled wells. In this case, further approximations are needed to guarantee a solution that converges to the optimum. On the same line, a possible solution would be to split the original network in subnetworks (clusters) and run an exact search algorithm in each cluster separately.

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