Modeling coupled heat and mass transfer for convection cooking of chicken patties

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Abstract

A 2-D axisymmetric finite element (FE) model was developed to simulate coupled heat and mass transfer during convection cooking of regularly shaped chicken patties, given actual transient oven air conditions. Transient temperature and moisture distributions inside the product were predicted, and 98 chicken patties were cooked in a convection oven for model validation. The predicted transient center temperature under various cooking conditions had an error of 3.8–5.7°C, as compared to experimental data. The prediction error for the cooking yield was 1.2%. For the same cooking conditions, the prediction error would increase to 5.9–12.9°C if mass transfer were ignored in simulation. The best prediction was obtained when both thermal conductivity and specific heat were modeled as state-dependent functions in the simulation. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Finite element modeling; Convection; Cooking; Yield; Poultry

Nomenclature

- \( A \) area, mm²
- \( c_T \) specific heat capacity, J/(kg °C)
- \( c_m \) specific moisture capacity, kg moisture/kg meat
- \( D_m \) moisture diffusivity, m²/s
- \( h_T \) convective heat transfer coefficient, W/(m² °C)
- \( h_m \) convective mass transfer coefficient, kg/(m² s)
- \( k_m \) moisture conductivity, kg moisture/(m s)
- \( k_T \) thermal conductivity, W/(m °C)
- \( L \) number of samples
- \( m, m_0 \) moisture content at time \( t \) and time 0, respectively, decimal wet basis
- \( m_e \) equilibrium moisture content of air, decimal wet basis
- \( m_1 \) moisture content at 4 nodes of an element, decimal wet basis
- \( n \) gradient normal to surface
- \( N_1 - N_4 \) shape functions of finite elements, dimensionless
- \( r, z \) cylindrical coordinates, mm
- \( R_g \) gas constant (1.987 g cal/g mol K)
- \( RH \) relative humidity, decimal
- \( t \) time, s
- \( \Delta t \) time step, s
- \( T, T_0 \) temperature at time \( t \) and time 0, °C
- \( T_a \) air temperature, °C
- \( T_1 - T_4 \) temperature at 4 nodes of an element, °C
- \( T_m \) temperature measured from experiment, °C
- \( T_p \) temperature predicted by the finite element simulation, °C
- \( \dot{T}, \dot{m} \) derivatives \( dT/dt \) and \( dm/dt \)
- \( \rho \) density, kg/m³
- \( \lambda \) latent heat of vaporization, J/kg
- \( \Gamma \) boundary

1. Introduction

In the poultry industry, fully-cooked products are a rapidly increasing portion of total product sales, with pre-cooked, refrigerated products projected to account for 80% of the industry's growth by the year 2005 (Neff, 1997). Therefore, thermal processes are increasingly important in determining the safety and quality and safety of retail products. The design and operation of these processes also influence the total cooking yield, which is an important economic factor for the industry. In order to analyze and improve these processes, considerable previous research has focused on simulation of heat penetration and moisture diffusion in meat products under various thermal conditions.

To predict heat penetration during thermal processing, heat transfer models have been widely employed for meat products. For example, temperature history and distribution in cooking, roasting, or frying, were simulated for chicken legs and pork ham (De Baerdemaeker,
Singh & Segerlind, 1977), beef patties (Dagerskog, 1979), beef loaf (Holtz & Skjoldebrand, 1986), and beef cuts (Califano & Zaritzky, 1993; Singh, Akins & Erickson, 1984). Noronha, Hendrickx, Suys and Tobback (1993) further used the simulated transient temperature history to optimize the quality retention during thermal processing of food products. In these previous studies, the heat transfer models were numerically solved either by the finite difference methods (Dagerskog, 1979; Singh et al., 1984; Holtz & Skjoldebrand, 1986; Noronha et al., 1993), finite element (FE) methods (De Baerdemaeker et al., 1977), or a boundary-fitting grid method in conjunction with a conservative control volume discretization (Califano & Zaritzky, 1993).

Similar methods have been used to solve the moisture diffusion problem. Mittal, Blaisdell and Herum (1982) investigated the mobility of moisture in meat emulsion sausage during cooking by means of a mass transfer model, numerically solved by the finite difference method. Dagerskog (1979) indirectly calculated internal mass transfer in beef patties during pan frying, based on the relationship between water retaining capacity of the minced meat and temperature. Singh et al. (1984) used a gas phase model to simulate evaporation of water from beef during oven roasting.

Limited reports have been found to describe simultaneous heat and mass transfer during cooking of meat products. Huang and Mittal (1995) developed a one-dimensional, coupled heat and mass transfer model for meatballs during forced convection baking, natural convection baking, and boiling. Mallikarjunan, Hung and Gundavarapu (1996) developed a two-dimensional simultaneous heat and mass transfer model to predict microwave cooking of cocktail shrimp simplified as a cylindrical circular slab geometry. The above models were all solved by finite difference methods.

Most of the previous research in this area has dealt with simplified geometries (sphere, infinite cylinder, infinite slabs) and assumed constant processing conditions and thermophysical properties. Consequently, while there does exist a body of relevant literature, there still remains a need for models that can be utilized for real-world processing applications. Therefore, this project is part of an overall program aimed at developing a coupled heat and mass transfer model to predict product temperature and yield, changes in physicochemical properties, and bacterial lethality, under actual convection cooking conditions in the poultry industry. Such a model should help to improve the design and operation of commercial cooking systems. As a first step towards this goal, this project had the following specific objectives:

- To develop a 2-D axisymmetric, coupled heat and mass transfer model using the FE method for convection cooking of poultry products.

- To implement the model with actual cooking conditions and empirical, nonlinear thermal property models to simulate thermal processing of chicken patties.

- To validate the model for thermal processing of chicken patties in a small convection oven.

2. Materials and methods

2.1. Coupled heat and mass transfer model

During convection cooking, heat is transferred mainly by convection from air to the product surface, and by conduction from the surface toward the product center. Meanwhile, moisture diffuses outward toward the product surface, and is vaporized. To simulate simultaneous heat and moisture transfer in poultry products during convection cooking, the following assumptions were made: there is no crust formation or shrinkage; mass transfer is only single species (i.e., only water); moisture diffuses to the product surface, and is evaporated only at the surface.

Based on the above assumptions, the general heat and mass transfer equations, boundary conditions and initial conditions for the 2-D axisymmetric problems were as follow:

heat transfer

\[
\rho \frac{\partial T}{\partial t} = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{k_r}{c_r} \frac{\partial T}{\partial r} \right) \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right),
\]

moisture transfer

\[
\rho \frac{\partial m}{\partial t} = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{k_m}{c_m} \frac{\partial m}{\partial r} \right) \right) + \frac{\partial}{\partial z} \left( k_m \frac{\partial m}{\partial z} \right),
\]

initial conditions

\[T(r, Z)|_{z=0} = T_0, \quad m(r, Z)|_{z=0} = m_0,\]

boundary conditions

\[k_r \frac{\partial T}{\partial n} = h_T(T_s - T) + D_m \rho e \frac{\partial m}{\partial n} \quad \text{(at the surface)},\]

\[k_m \frac{\partial m}{\partial n} = h_m(m_e - m) \quad \text{(at the surface)},\]

where the symbols are defined in the nomenclature. These models show uniform initial temperature and moisture distribution (Eq. (3)), convective heat transfer at the surface with latent heat loss due to evaporation (Eq. (4)), and convective mass transfer at the surface (Eq. (5)).

The FE method (Reddy, 1984) was used to solve the above equations. The cross section of a chicken patty was divided into 4-node linear quadrilateral elements. Within each element, the temperature and moisture at
any point were expressed in terms of the temperature or moisture at its four nodes

\[ T^{(e)} = \sum_{i=1}^{4} N_i T_i, \quad m^{(e)} = \sum_{i=1}^{4} N_i m_i. \]  

(6)

The coupled heat and mass transfer model, Eqs. (1)–(5), was numerically solved for temperature and moisture in all the nodes by

\[ \begin{bmatrix} C_T & 0 \\ 0 & C_m \end{bmatrix} \begin{bmatrix} \dot{T} \\ \dot{m} \end{bmatrix} + \begin{bmatrix} K_T & K_{Tm} \\ K_{Tm} & K_m \end{bmatrix} \begin{bmatrix} T \\ m \end{bmatrix} = \begin{bmatrix} F_T \\ F_m \end{bmatrix} \]  

(7)

in which \( C_T, C_m, K_T, K_m, \) and \( K_{Tm} \) are global matrices of size total node by total node and \( F_T, F_m, T \) and \( m \) are global vectors of size total node by 1. These matrices and vectors are assembled from the following element matrices \( C_T^{(e)}, C_m^{(e)}, K_T^{(e)}, K_m^{(e)}, \) and \( K_{Tm}^{(e)} \) (matrix size of \( 4 \times 4 \)) and element vectors \( F_T^{(e)}, F_m^{(e)} \) (size \( 4 \times 1 \)):

\[ C_T^{(e)} = \int_A \rho c_T [N]^T [N] \, dA, \]  

(8)

\[ C_m^{(e)} = \int_A \rho c_m [N]^T [N] \, dA, \]  

(9)

\[ K_T^{(e)} = \int_A k_T [B]^T [B] \, dA + \int_f h_T [N]^T [N] \, d\Gamma, \]  

(10)

\[ K_m^{(e)} = \int_A k_m [B]^T [B] \, dA + \int_f h_m [N]^T [N] \, d\Gamma, \]  

(11)

\[ K_{Tm}^{(e)} = \int_f \lambda h_m [N]^T [N] \, d\Gamma, \]  

(12)

\[ \begin{bmatrix} F_T^{(e)} \\ F_m^{(e)} \end{bmatrix} = \int_f (h_T T_s + \lambda h_m m_s) [N]^T \, d\Gamma, \]  

(13)

\[ \begin{bmatrix} F_T^{(e)} \\ F_m^{(e)} \end{bmatrix} = \int_f h_m m_s [N]^T \, d\Gamma, \]  

(14)

\[ [B] = \begin{bmatrix} \frac{\partial N}{\partial x} \\ \frac{\partial N}{\partial y} \end{bmatrix}, \]  

(15)

where \([N]\) is a \( 1 \times 4 \) vector containing element shape functions, and \([B]\) is a \( 2 \times 4 \) transpose matrix. The FE solution of this transient field problem was further solved by the Crank–Nicholson central difference method (Reddy, 1984; Mallikarjun & Mittal, 1994) as

\[ \begin{bmatrix} \hat{K} \end{bmatrix} \begin{bmatrix} T \\ m \end{bmatrix}_{n+1} = \{F\}, \]  

(16)

where

\[ \hat{K} = \begin{bmatrix} C_T & 0 \\ 0 & C_m \end{bmatrix} + 0.5\Delta t \begin{bmatrix} K_T & K_{Tm} \\ K_{Tm} & K_m \end{bmatrix}. \]  

(17)

The above equations were coded in MATLAB version 4.2c1 (MathWorks, Natick, Mass).

2.2. Simulation of chicken patties during cooking

The FE model was applied to simulate convection cooking of chicken breast meat patties. The patties were 1.0 cm thick and 6.2 cm in diameter, and contained about 78% water (wet basis), 95% protein (dry basis) and 0.15% lipids (dry basis) (Murphy, Marks & Marcy, 1998). Due to the axisymmetry of the patty (Fig. 1), only one quarter of a planar intersection was needed for simulation. The representative intersection was discretized into 32 4-node rectangular elements (Fig. 2). The coordinate origin represented the center of the chicken patty. The boundaries \( \Gamma_2 \) and \( \Gamma_3 \) represented the surface of the chicken patty, while the boundaries \( \Gamma_1 \) and \( \Gamma_4 \) represented the symmetry lines inside the chicken patty, which had boundary conditions of

\[ \frac{\partial T}{\partial n} = 0, \quad \frac{\partial m}{\partial n} = 0 \quad (\Gamma \subseteq \Gamma_1, \Gamma_4). \]  

(19)

The cooking model was run with several different combinations of thermophysical properties. In preliminary runs, only constant values (Table 1) were used. The convective heat transfer coefficient \( h_T \) was estimated to minimize the prediction error and was based on Huang and Mittal (1995) for low air velocity. Because they are not well documented in the literature, the specific moisture capacity \( c_m \) and convective mass transfer coefficient \( h_m \) used by Scheerlinck, Nicolai, Verboven and De Baerdemaeker (1996) for other food materials were used for the simulation. Unless otherwise stated, subsequent runs used the following empirical models for specific heat and thermal conductivity, which were reported in and derived from Murphy et al. (1998) and Murphy and Marks (1998), respectively

\[ c_T = 3017.2 + 2.05 T + 0.24 T^2 + 0.002 T^3, \]  

(20)

\[ k_T = 0.194 + 0.436 m. \]  

(21)

![Fig. 1. Geometry of a chicken patty.](image)
Actual oven air temperature data recorded in cooking tests were used in the FE simulation as transient boundary conditions. The relative humidity (RH) of the oven air was estimated from psychrometric relationships (ASAE Standards, 1995), by assuming that the ambient room air was heated to the oven temperature without addition of any water to the air. Subsequently, the equilibrium moisture content ($m_e$) of the air for meat products was estimated by the following function (Huang & Mittal, 1995)

$$RH = \exp\left(\frac{-5222.47}{K_{\tau}T_{\alpha}} m_e^{-1.0983}\right),$$

where $m_e$ was in decimal dry basis. From this procedure, an estimated $m_e$ value of 0.02 (wet basis) was used in all of the simulations.

2.3. Experimental cooking tests

To validate the above simulations, chicken patties were cooked by convection in a small convection oven (Montgomery Ward model 8287). Prior to cooking, each frozen patty was sealed in a polyethylene bag to prevent water loss, thawed overnight at 3°C, and then stabilized for 1–2 h at room temperature (about ~22°C). This procedure ensured uniform temperature and moisture distribution inside each sample before cooking.

Seven cooking air temperatures (135, 149, 163, 177, 191, 204 and 218°C) and seven endpoint meat center temperatures (50, 55, 60, 65, 70, 75 and 80°C) were selected for cooking tests. For each air/endpoint temperature combination, two chicken patties were cooked. One T-type thermocouple was inserted to the center of a patty to measure its center temperature and another thermocouple was fixed near the patty to measure the cooking air temperature. The thermocouples were connected to a data acquisition system outside the oven, with temperatures recorded every 10 s. Patties were weighed prior to and after cooking, with an accuracy of 0.01 g. Cooking yield was the weight of the cooked patty expressed as a percentage of its original weight before cooking.

3. Results and discussion

3.1. Simulated temperature and moisture content distributions

Prior to utilizing the model, the heat transfer code was validated against an analytical solution for the same geometry, constant material properties (Table 1), and air temperature of 218°C. These results, utilizing the truncated infinite series solution commonly represented in the Heisler charts (Holman, 1986), yielded surface and center temperatures within 0.8°C of the numerical solution. The model and corresponding codes were also evaluated by varying the number of elements and the size of the time step for simulation under a target air temperature of 218°C. Increasing the FE nodes from 32 to 64 caused a maximum variation of 0.03°C for the center temperature and 0.53% for the cooking yield. Decreasing the time step from 10 to 2 s caused a maximum variation of 0.41°C for the center temperature and 0.09% for the cooking yield.

For illustration purposes, the cooking model was used to predict temperature and moisture distributions inside a chicken patty cooked to a center temperature of 70°C under four different air conditions. The product temperature (Fig. 3) varied more along the radial direction than along the z-axis. Under a target air temperature of 135°C, 850 s was required to heat the patty center to 70°C, and the temperature gradient was ~14°C from the center to the outside edge. In contrast, under a target air temperature of 218°C, the cooking time to 70°C was reduced to 380 s, and the predicted radial temperature gradient increased to ~35°C. Therefore, an increase in air temperature (or heating rate) yielded a less uniform temperature distribution within the patty.
As expected, the simulated moisture distribution (Fig. 4) showed an opposite trend to the temperature gradient, with the highest moisture content in the center of the patty and the lowest at the patty edge. The cooking yield for the entire patty was computed from the change in mass average moisture content. Under a target air condition of 218°C for 380 s, the moisture gradient was 0.28 (from 0.5 to 0.78), corresponding to a cooking yield of 90%. However, at 135°C for 850 s, the gradient increased to 0.48 (from 0.3 to 0.78), corresponding to a cooking yield of 85%.

3.2. Experimental validation of simulation results

For model validation, the measured and simulated center temperatures of chicken patties cooked under four target air temperatures were compared (Fig. 5). The thermal properties models (Eqs. (20) and (21)) were included in the FE model, and the measured transient oven temperature data were used for all these simulations. Additionally, the measured and simulated cooking yields for 98 samples cooked under all the described conditions were compared (Fig. 6). For each criterion, the prediction error was determined by the standard error (SE) or prediction

\[ \text{SE} = \sqrt{\frac{\sum (Y_p - Y_m)^2}{L - 1}}, \] (23)

where \( Y_p \) and \( Y_m \) are the predicted and measured values, respectively, and \( L \) was the number of observations for each calculation (38–85 for each temperature history and 98 for the cooking yield data). Under the four measured cooking conditions, the prediction error for the transient center temperature ranged from 3.7°C to 5.5°C. For all 98 samples, the prediction error for the cooking yield was 1.2%. Therefore, the simulated results agreed reasonably well with the measured results.

3.3. Major factors affecting simulation results

As expected, coupling the mass transfer model to the heat transfer model influenced the predicted temperature results (Fig. 7). The center temperatures from the coupled simulation were always lower than those from
the uncoupled simulation. For the same four cases analyzed in the previous section (Fig. 6), the SE of prediction increased to 6.2–13.4°C when the mass transfer model was not included.

The coupled model included the effect of mass transfer on heat transfer, but did not include the effect of heat transfer on mass transfer. It was, therefore, a one-way coupled model. Two-way coupled heat and mass transfer models (Husain, Chen & Clayton, 1973, Scheerlinck et al., 1996) might better describe the simultaneous heat and mass transfer phenomena. However, the major difficulties in using these models would be the additional parameters (e.g., the ratio of vapour diffusion coefficient and the thermo-gradient coefficient) that would be necessary, but which are not well documented in the literature.

Even in the one-way coupled model, thermal properties significantly affect simulation results. Thermal processing involves not only simultaneous heat and mass transfer, but also physicochemical reactions, such as protein denaturation. Consequently, the thermal properties change during processing. To investigate the effects on model accuracy (SE), four different combinations of constant or variable thermal properties were compared (Table 2). Case 1 included constant thermal property values as listed in Table 1. Case 2 substituted the thermal conductivity function (Eq. (21)), derived from the data of Murphy and Marks (1998), which was applied at each node of the FE model. Case 3 instead substituted the nonlinear specific heat capacity function (Eq. (20)), derived by Murphy et al. (1998), to each node. Lastly, case 4 included both property functions (Eqs. (20) and (21)). Either the $k_T$ or $c_T$ function (case 2 and 3) alone improved the simulation accuracy (Table 2) over the constant values (case 1). The best prediction was obtained when both the $k_T$ and $c_T$ functions were included in the simulation. Good state-dependent models for other thermophysical properties (e.g., specific moisture capacity, moisture conductivity, convection mass transfer coefficient) would be expected to further improve the simulation results. However, these functions are difficult to determine, and are not yet readily available.

Fig. 5. Transient center temperature and cooking yield for chicken patties cooked in a small convection oven. The standard errors of prediction for center temperature were (a) 5.5, (b) 4.0, (c) 5.2 and (d) 3.7°C.
Lastly, the invalidity of model assumptions can always influence prediction results. For example, during the cooking tests, a small amount of shrinkage was observed with increasing cooking time and temperature. The current model ignored the product shrinkage, which likely introduced some error.

4. Conclusions

A FE model, which considered one-way coupled simultaneous heat and mass transfer, was established to describe convection cooking of chicken patties. Given measured cooking air conditions as inputs, the model was used successfully to predict transient temperature and moisture distributions inside the product, as well as transient cooking yield of chicken patties during cooking. The prediction accuracy was improved by including state-dependent functions for thermal properties at each node in the model. With future incorporation of physicochemical models and bacterial lethality models, such a model could be further used to optimize cooking conditions, procedures and facilities for the control of food quality and safety.

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Table 2
Prediction errors for center temperature under the target air temperature of 218°C

<table>
<thead>
<tr>
<th>Case</th>
<th>Property source</th>
<th>SE of prediction (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Book values</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>$k_T = f(m)$</td>
<td>4.4</td>
</tr>
<tr>
<td>3</td>
<td>$c_T = f(T)$</td>
<td>4.3</td>
</tr>
<tr>
<td>4</td>
<td>$k_T = f(m)$, $c_T = f(T)$</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Fig. 6. Observed and predicted cooking yield versus cooking time ($n = 98$).

Fig. 7. Example transient center temperature, comparing coupled heat and mass transfer model, a simple heat transfer model and corresponding experimental data.

References


