Convex Analysis, Sect. 3.1

Exercise 1

To prove positive homogeneity, show that $f(0) = 0$. Further, for all $\lambda \in \mathbb{R}_{++}$, $x \in E$: $f(x) = f(\lambda^{-1}\lambda x + 0)$.

Exercise 3

Represent the subdifferential as an intersection of closed and convex sets.

Exercise 6

Revisit Exercise 7 in Section 2.1. Note that the function may assume infinite values now (though we select $\bar{x} \in \mathrm{dom} f$).

Exercise 9

Fan's inequality is the central tool. Showing the inclusion $\partial\lambda_1(0) \subset \{\, Y \in \mathcal{S}^n_+ \mid \mathrm{tr} Y = 1 \,\}$ requires selecting "creative" directions $X \in \mathcal{S}^n$. Experiment with matrices, which admit simultaneous ordered spectral decomposition with the subgradient.