TMA 4320 – PHYSICS PROJECT 2

This exercise considers the motion of planets in the solar system. The planetary motion is governed by the Newtonian force law of gravity (see the Appendix for planetary motion, pdf slides). Notice that all planets in the solar system move approximately in the same plane, which means that two-dimensional simulations will be sufficient. In the first two parts of the project (topics 1 and 2), the motion of a single planet is investigated. After that, we shall consider the motion of an orbiting satellite (topic 3), and, finally we'll consider the three-body problem for Earth and Mars (topic 4). For numerical values of the gravitational constant, planetary masses, etc., please take a look at the Appendix.

1. Planetary motion.

- First, design a planetary orbit programme based on the **Runge-Kutta (RK) algorithm (4th order)** which can reproduce the **trajectory of Earth for fixed Sun** starting from from time $t_{\min}$ and and ending at $t_{\max}$ using a time step τ (set by the user). The physical units of your programme should be the astronomical units (AU, years and etc.). For Earth, it takes 1 year to complete a full revolution which provides us a (starting) velocity of 2π (AU/yr). Record the position (*x*- and *y*-coordinates), velocity, kinetic energy, potential energy and total energy as functions of time. *Include also the simpler Euler-Cromer (EC) algorithm in your code for comparison purposes.*
- It is important to perform a **convergence test**, and you should consider the effect of the time step τ on your trajectory. What would be good initial guesses for RK and EC? Why? For one particular planet (e.g., the Earth), optimize τ using the requirement of total energy conservation for both RK and EC. To this end, calculate the energy change over one orbit and plot it as a function of τ. What type of behavior do you expect? Choose an appropriate way of plotting the result. Compare between the Runge-Kutta and Euler-Cromer algorithm throughout.
- Demonstrate ***Keplers's third la*w** for all planets with nearly circular orbits (for parameters see the Appendix). Think about how to choose the initial conditions to obtain circular orbits.

2. Perihelion of Mercury á la Einstein.

We shall consider the stability of planetary orbits by considering the **prediction by Einstein** for the precession of the perihelion of Mercury based on the ***General Theory of Relativity*** (see the Appendix). The observed precession is of the order 566 arcseconds per century (0.1572 degrees), and it is mostly caused by the gravitational effect of other planets (as numerically calculated by early astronomers). However, there was still a peculiar contribution of 43 arcseconds per century which remained unexplained. Based on his ideas on gravitation, Einstein provided an analytical solution for this problem.

- Start the simulation by considering the **case without precession**. Keep the Sun fixed and place Mercury on an elliptical orbit by starting the simulation from perihelion (nearest point to the Sun) where the velocity is at its maximum. As before, record the position, velocity, kinetic energy, potential energy and total energy as functions of time, and make sure that you are using a proper time step. *You may use the RK algorithm solely from now on.*

- The force law with **Einstein's correction** states that the gravitational force becomes slightly modified from the Newtonian inverse-square form

$$F_G \approx \frac{GM_S M_M}{r^2}\left(1+\frac{\alpha}{r^2}\right)$$

and leads to a slow precession (rotation) of the perihelion for repeated orbits. Modify your program to investigate this effect. The value of $\alpha = 1.1 \times 10^{-8}$ AU^2 derived by Einstein is very small. What can be done instead? Calculate for several larger values of α and use your results for estimating the correct value of precession. [Hint: You should obtain the correct value.]

3. Earth satellite orbits.

A communication satellite is carried by a Space Shuttle into low earth orbit (LEO) at an altitude of 322 km and is to be transferred to a geostationary orbit (GEO) at 35 680 km. The satellite weights 720 kg. The gravitational constant is $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

- Now, apply your planetary orbit programme for the **satellite trajectory in LEO** where you keep the Earth fixed this time. Note that the physical units of your programme should be standard (meters, seconds, etc.) rather than astronomical ones and you need to determine an appropriate time step τ for this problem (for RK). As before, record the position, velocity, kinetic energy, potential energy and total energy as functions of time. Remember that Earth has a radius itself!
- By testing with different velocity changing profiles including a limited (small) number of discrete velocity changes (additions), demonstrate that **the most fuel-efficient maneuver from LEO to GEO** is to apply two velocity increases (Δv_1 and Δv_2) at two phases (t_1 and t_2) of the transfer. You can focus on the net velocity increase ($\Sigma \Delta v_i$) since the fuel consumption is directly proportional. This is due to the increased linear momentum of the satellite during the release of exhaust gases while burning up the fuel (principle: conservation of linear momentum; we'll neglect changes in the satellite overall weight here). Which are these two phases and how does the velocity vector change in each case? How much is the net velocity increase? What is the time between t_1 and t_2 in this process. Demonstrate these things numerically! [Hint: Hohmann transfer]

4. Three-body problem: Earth and Mars.

- Next, modify your program from topic 1 (planetary motion) so that it includes also the gravitational **interaction between Earth and Mars**. See the Appendix on planetary motion for more details (example case therein: Earth and Jupiter). The physical units of your programme should be the astronomical units, again. Record the position, velocity, kinetic energy, potential energy and total energy of both planets as functions of time. Demonstrate that the corresponding trajectories for both planets are stable.
- **Hypothetical test**: Assume that either Earth or Mars has the **mass of Jupiter**. How do the trajectories look like? And what if they both had the mass of Jupiter? You may scale up the masses so long that funny things start to occur.

HAPPY COMPUTING…