Initial Value Problem

$$\begin{cases} y'(t) = f(t, y(t)), & t > a, \\ y(a) = y_a. \end{cases}$$

Existence/uniqueness of solutions

Theorem 6.2 (Picard-Lindelöf):
Assume

- $|f(t, y_1) - f(t, y_2)| \leq L|y_1 - y_2|$, $\forall y_1, y_2 \in [\alpha, \beta]$, $\forall t \in [a, b]$
  (f - uniformly Lipschitz in y on $S = [a, b] \times [\alpha, \beta]$)
- $y_a \in [\alpha, \beta]$

Then $\exists c > a$: IVP solution exists & unique on $[a, c]$

Remember: quite possibly $c < b$!

Continuous dependence on initial data

Theorem 6.3:
Assume

- $|f(t, y_1) - f(t, y_2)| \leq L|y_1 - y_2|$, $\forall y_1, y_2 \in [\alpha, \beta]$, $\forall t \in [a, b]$
  (f - uniformly Lipschitz in y on $S = [a, b] \times [\alpha, \beta]$)
- $y(t)$ solves IVP on $[a, b]$ with $y(a) = y_a$
- $z(t)$ solves IVP on $[a, b]$ with $z(a) = z_a$
- $y(t) \in [\alpha, \beta]$, $z(t) \in [\alpha, \beta]$ for all $t \in [a, b]$

Then:

$$|y(t) - z(t)| \leq e^{L(t-a)}|y_a - z_a|, \qquad \forall t \in [a, b].$$