TMA4329 Intro til vitensk. beregn.
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ving 7

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[S]=T. Sauer, Numerical Analysis, Second International Edition, Pearson, 2014

"Teorioppgaver"

1 Oppgave 6.4.4, (a), (b) s. 321, [S]

2 Oppgave 6.4.5, s. 321, [S]

3 Oppgave 6.4.6, s. 321, [S]

4 Oppgave 6.4.7, s. 321, [S]

5 Oppgave 6.6.1 (a), (b), s. 335, [S]

6 Oppgave 6.6.4, s. 335, [S]

7 Consider the initial value problem

$$\begin{aligned} y' &= \lambda y, \qquad t > 0, \\ y(0) &= y_0, \end{aligned}$$

where $\lambda \in \mathbb{C}$. Its solution is $y(t) = y_0 \exp(\lambda t)$.

a) Suppose that we use a numerical method (such as e.g. forward Euler or explicit trapezoid) to solve this problem starting from a point $w_0 = y_0$. *The stability region* for the method is a set of points $z = \lambda h$ in the complex plane, such that the numerical solution $(w_0, w_1, \dots)$ stays *bounded* (i.e., $\exists C > 0 : \forall i, |w_i| \leq C$). Find the stability region for (1) implicit (backward) Euler method, defined by the formula $w_{i+1} = w_i + hf(t_{i+1}, w_{i+1})$, see p. 333 in [S]; (2) implicit Trapezoid method defined by the formula $w_{i+1} = w_i + h/2[f(t_i, w_i) + f(t_{i+1}, w_{i+1})]$.

b) Let $\lambda = j\omega$, where $j^2 = -1$ and $\omega > 0$. Show that the implicit trapezoid method matches the *amplitude* of the solution exactly, that is, $|w_i| = |y(t_i)|$, for all $i = 1, 2, \dots$

"Computeroppgaver"

8 Oppgave 6.4.12, s. 322, [S]

9 Oppgave 6.6.2, s. 336, [S].