TMA4320 Intro til vitensk. beregn. V2016

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ving E02

Here is a collection of some exam questions taken from the prevous exams in TMA4215 and MA2501 (sometimes modified), which are pertinent to TMA4320.

This particular set of questions is somewhat heavy on ODEs/quadratures/interpolation; to the contrary, solving equations and DFT is underrepresented.

≈Problem 2, Exam TMA4215, 13.12.2014

a) The value of a function $f(x)$ is known at the points:

x_i	0.0	2.0	3.0
$f(x_i)$	2.0	12.0	20.0

Find the polynomial $p(x)$ of lowest possible order interpolating $f(x)$ in these points.

b) Find the polynomial $q(x)$ of lowest possible order, which satisfies the interpolation condition above *in addition* to the condition $f(1.0) = 6.0$.

≈Problem 3, Exam TMA4215, 13.12.2014

a) Determine the values c_j, $j = -1, 0, 1, 2$, such that the quadrature rule $Q(f) = c_{-1}f(-1) + c_0 f(0) + c_1 f(1) + c_2 f(2)$ gives the correct value for the integral $\int_0^1 f(x)\,\mathrm{d}x$ when f is an arbitrary polynomial of degree 3.

b) Let us denote the composite trapezoid quadrature with m panels with T_m and similarly the composite midpoint rule with M_m. Show that $M_m = 2T_{2m} - T_m$.

c) Construct polynomials of order 0, 1, 2, which satisfy the orthogonality condition $\int_{-1}^1 P_i(x)P_j(x)\,\mathrm{d}x = 0$ when $i \neq j$. Which quadratures have nodes at the roots of such polynomials and why?

≈Problem 4, Exam TMA4215, 13.12.2014

Consider the following initial value problem:

$$y'(t) = f(t, y), \quad t_0 < t < T, \quad y(t_0) = y_0.$$

Let us consider the following numerical method for this equation:

$$y_{n+1} = y_n + \frac{h}{2}(k_1 + k_2),$$

where

$$k_1 = f(t_n, y_n), \quad k_2 = f(t_n + h, y_n + hk_1).$$

Using a Taylor series expansion of $f(\cdot,\cdot)$ and $y(\cdot)$, find the local truncation error of this numerical method.

≈Problem 3, Exam TMA4215, 17.08.2011 Consider the initial value problem for a second order ODE:

$$y''(t) = y'(t)y(t), \quad y(0) = 1, \quad y'(0) = 0.5.$$

a) Reformulate the problem into a system of first order ODEs.

b) Apply one step of explicit trapezoid method to this system to find the numerical approximation of $y(0.1)$ and $y'(0.1)$.

≈Problem 1, Exam TMA4215, 02.12.2013 **a)** Set up the interpolation polynomial $p(x)$ in Lagrange form for the following data set:

x_i	1	3	4
y_i	3	1	8

b) The equation

$$2x = e^x - 1,$$

has a solution at $x^* \approx 1.25643$. Will the iteration scheme

$$x_{k+1} = \frac{e^{x_k} - 1}{2}$$

converge to x^* if x_0 is sufficiently close to it?

c) Find the degree of accuracy (precision) for the quadrature formula $Q(f)$:

$$\int_0^1 f(x)\,\mathrm{d}x \approx Q(f) = \frac{3f(1/3) + f(1)}{4}.$$

d) An ordinary differential equation $y'(t) = f(t, y(t))$, $y(t_0) = y_0$ is solved by some Runge-Kutta method, using stepsize $h = (t_{\text{end}} - t_0)/N$. The following table lists the global error $e_N = y_{t_{\text{end}}} - y_N$ for different values of N:

N	20	40	80	160	320
e_N	$1.93 \cdot 10^{-6}$	$2.15 \cdot 10^{-7}$	$2.54 \cdot 10^{-8}$	$3.08 \cdot 10^{-9}$	$3.80 \cdot 10^{-10}$

From this experiment, what would you expect the order of the method to be?

≈Problem 1, Exam TMA4215, 19.12.2009 A *non-composite* Simpson's rule is given by

$$\int_a^b f(x)\,\mathrm{d}x = \frac{1}{3}\frac{b-a}{2}[f(a) + 4f((a+b)/2) + f(b)] - \frac{1}{90}\left(\frac{b-a}{2}\right)^5 |f^{(4)}(\xi)|,$$

where $\xi \in [a, b]$.

a) Find an approximation to the integral

$$\int_1^2 x^2 \ln x \,\mathrm{d}x$$

using Simpson's composite formula based on 4 panels ($h = 0.25$).

b) Find an upper limit for the error for the integral in **a)**.

≈Problem 3, Exam TMA4215, 19.12.2009 **a)** The value of a function $f(x)$ is known in the points

x_i	0.00	0.60	1.00
$f(x_i)$	0.00	0.75	0.50

Find the polynomial $p(x)$ of lowest possible order interpolating $f(x)$ at these three points.

b) Find the polynomial $q(x)$ of which satisfies the interpolation conditions given above *in addition to* the condition $q'(0.6) = f'(0.6) = 0.5$ using the following strategy: let $q(x) = p(x) + r(x)$. Knowing the roots of $r(x)$, we can then scale $r(x)$ in such a way that the condition for the derivative for $q(x)$ is satisfied.

≈Problem 4, Exam TMA4215, 19.12.2009 Consider the initial value problem for a second order ODE:

$$y''(t) = f(t, y(t)), \quad y(t_0) = y_0, \quad y'(t_0) = z_0.$$

a) Reformulate the problem into a system of first order ODEs.

b) Show that two steps of explicit Euler's method with constant steplength h applied to the first order system are equivalent to using the following method:

$$y_{n+1} - 2y_n + y_{n-1} = h^2 f(t_{n-1}, y_{n-1}).$$