“Teorioppgaver”

1. Oppgave 10.1.1

2. Oppgave 10.1.7

3. Prove the following identity (Parseval’s theorem for DFT): Let $y^{(1)} = \text{DFT}(x^{(1)})$ and $y^{(2)} = \text{DFT}(x^{(2)})$. Then $\sum_{j=0}^{n-1} x_j^{(1)} \overline{x_j^{(2)}} = \sum_{k=0}^{n-1} y_k^{(1)} \overline{y_k^{(2)}}$.

   Hint: $y^{(1,2)} = F_n x^{(1,2)}$.

4. Prove the following identity (circular shift theorem for DFT): Let $\tilde{x}_j = \exp(i2\pi jm/n)$, where $i^2 = -1$ and $m$ is an integer. Let $y = \text{DFT}(x)$, and $\tilde{y} = \text{DFT}(\tilde{x})$. Show that $\tilde{y}_k = y_{k-m}$, where the subscript $k - m$ is understood modulo $n$ (in other words, the sequence $y$ is assumed to be periodically repeating with period $n$).

“Computeroppgaver”

5. Implement Cooley–Tukey’s algorithm for computing DFT:

   function [y,nop] = myfft(x)
   % Implementation of FFT
   %
   % Input: x, supposed to be a vector of length $2^p$
   %
   % Output: y: FFT(x)
   %
   % nop: number of operations required
   %

   Verify it against Matlab’s FFT (recall that Matlab uses a different scaling of DFT). Check that the number of operations needed by Cooley–Tukey’s algorithm scales as
\( O(N \log(N)) \) by plotting the number of iterations vs. \( N \) and \( N \log(N) \) on a log-log plot for a range of \( N = 2^p \).