Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

Faglig kontakt under eksamen:
Peter Lindqvist tlf. 73 59 35 29

Eksamen i TMA4305 Partielle Differensialligninger

Partial Differential Equations
English
Fredag 4. juni 2010
9:00 – 13:00
Sensurdato: 25.06.2010

Hjelpemidler Godkjent kalkulator.
Et A4 ark stemplet av IMF med valgfri påskrift av studenten.

Oppgave 1

Consider the equation

$$u_t + uu_x = 0 \quad (-\infty < x < \infty,\ t > 0)$$

with the initial condition

$$u(x,0) = \begin{cases} 0, & \text{when } x < 0 \\ x, & \text{when } x \geq 0 \end{cases}$$

Find the solution.

Oppgave 2

Describe the Perron Method for the Laplace Equation (Hints: Subharmonic function, lower class, lower solution, barrier function). No Proofs are required!

Oppgave 3

Let $u \epsilon C^2(\overline{\Omega_T})$ be a solution to the dissipative wave equation

$$u_{tt} - c^2 \Delta u + u_t = 0$$

in $\Omega_T = \Omega \; x \; (0, T)$, where Ω denotes a bounded domain in $\mathbf{R}^3$. Assume that $u(x,t) \equiv 0$, when $x \epsilon \partial\Omega$ and $t \geq 0$. Show that the energy

$$E(t) = \frac{1}{2} \iiint_{\Omega} (u_t^2 + c^2 |\nabla u|^2) dxdydz$$

is decreasing, $0 \leq t \leq T$. Hint: $E'(t)$.

Oppgave 4

Suppose that $u \epsilon C^2(\overline{\Omega})$ is a solution to the Allen-Cahn equation

$$\Delta u = u(u^2 - 1)$$

in Ω with boundary values 1/2. Here Ω is a bounded domain in the plane. Prove that $u \leq 1$ in Ω.

Oppgave 5

Solve the initial value problem

$$\begin{cases} u_{tt} = u_{xx} + u_{yy} + u_{zz} \\ u(x,y,z,0) = x^2 + y^2 \\ u_t(x,y,z,0) = 0 \end{cases}$$

Oppgave 6

Consider the variational integral

$$I(v) = \int_0^1 \int_0^1 \left[e^x \left(\frac{\partial v}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 \right] dxdy$$

for all functions $v \epsilon W^{1,2}(Q)$ with $v - f \epsilon W_0^{1,2}(Q), f(x,y) = x^2 + y$. Here Q is the square. Prove, that a minimizer u exists, i.e.

$$I(u) \leq I(v)$$

for all admissible v. Then write the Euler-Lagrange equation, assuming that u has continous second derivatives.