Assume that $f \in L^2(\Omega)$ and that $g \in C(\bar{\Omega}) \cap H^1(\Omega)$. Let $u \in H^1(\Omega)$ be a solution of

$$\begin{cases} \int_\Omega (\nabla u \cdot \nabla \phi + f\phi)\, d^n x = 0 \text{ for all } \phi \in C_0^\infty(\Omega), \\ u - g \in H_0^1(\Omega). \end{cases}$$

Show that the eqn also holds when $\phi \in H_0^1(\Omega)$. Then, prove that u is unique (if it exists).

In other words,

$$\begin{cases} \Delta u = f & \text{in } \Omega \\ u = g & \text{on } \partial\Omega \end{cases}$$

in a weak sense.