Sobolev-regularitet

Teorem: Dersom $m > k + \frac{n}{2}$, er $H^m(\mathbb{T}^n) \subset C^k(\mathbb{T}^n)$.

Bevis-skisse: $u \in L^2(\mathbb{T}^n) \Rightarrow u(x) = \sum_{j \in \mathbb{Z}^n} c_j e^{ij\cdot x}$ sum i L^2

$u \in H^m(\mathbb{T}^n)$: $D^\alpha u(x) = i^{|\alpha|} \sum_{j \in \mathbb{Z}^n} j^\alpha c_j e^{ij\cdot x}$ sum i L^2 $|\alpha| \le m$

$$\sum_{j \in \mathbb{Z}^n} |j|^{2m} |c_j|^2 < \infty$$

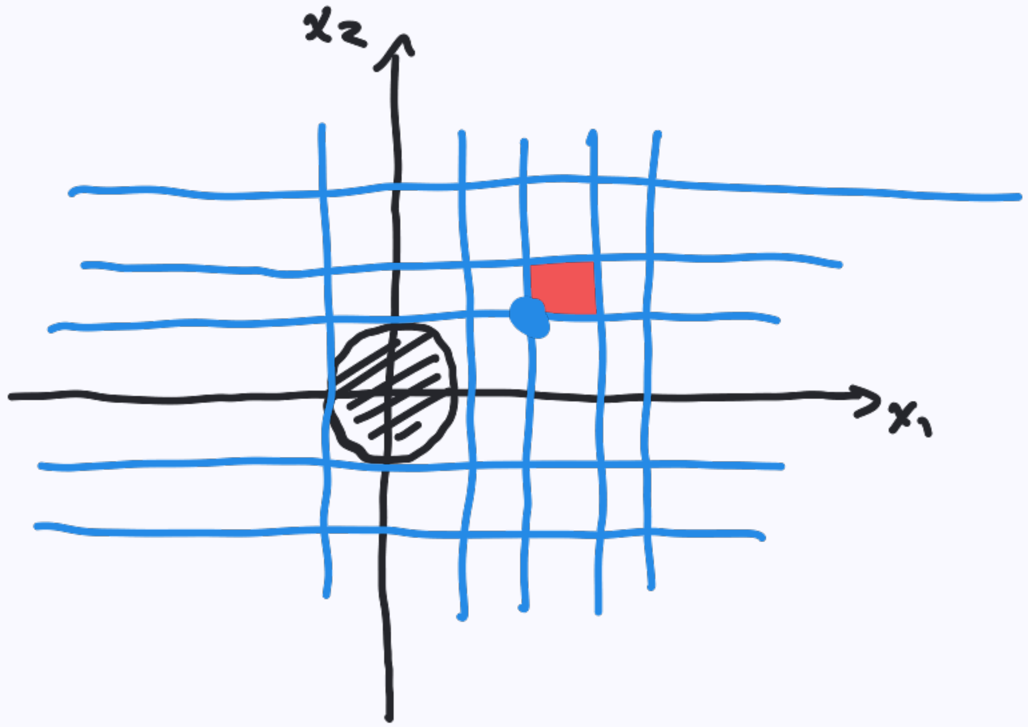
Mål: ønsker å vise at $\sum |j^\alpha c_j| < \infty$ dersom $|\alpha| \le k$

$$\sum_{j \in \mathbb{Z}^n} |j|^k |c_j| < \infty$$

$$\sum_{j \neq 0} |j|^{k-m} \cdot |j|^m |c_j| \le \left(\sum_{j \neq 0} |j|^{2(k-m)}\right)^{1/2} \underbrace{\left(\sum_j |j|^{2m} |c_j|^{2m}\right)^{1/2}}_{<\infty}$$

$s = 2(k-m) < \infty$

$$\int_{\mathbb{R}^n \setminus B^n} |x|^s \, d^n x \geq \sum_{\substack{j \in \mathbb{Z}^n \\ j \neq 0}} |j|^s \qquad (s<0)$$



$$|x| \geq |j| \Rightarrow |x|^s \leq |j|^s$$

$$\int_{\mathbb{R}^n \setminus B^n} |x|^s d^n x = A_n \int_1^\infty r^s \cdot r^{n-1} dr < \infty$$

$$\Leftrightarrow s+n-1 < -1 \Leftrightarrow s+n < 1$$

$$2(k-m)+n < 1$$

$$m < k + 2n$$