① $3u_y + u_{xy} = 0$

a) What type? (Elliptic, para...)

b) General solution?

c) Discuss the Cauchy problem

$$\begin{cases} u(x,0) = e^{-3x} \\ u_y(x,0) = 0 \end{cases}$$

② Prove <u>Harnack's</u> <u>Principle</u>: If

$$0 \le h_1 \le h_2 \le h_3 \le \cdots$$

is an increasing sequence of harmonic functions in Ω, then <u>either</u> $h(x) = \lim_{j\to\infty} h_j(x) \equiv \infty$ <u>or</u> h is harmonic in Ω.

<u>Hint</u>: If $D \subset\subset \Omega$, then

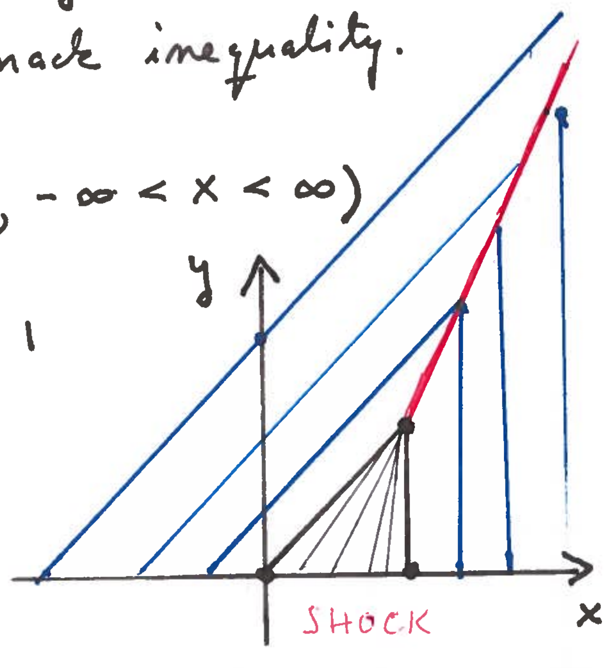
$$\max_{\bar{D}} h_j \le C_K \min_{\bar{D}} h_j,$$

a consequence of the Harnack inequality.

③ $uu_x + u_y = 0 \quad (y > 0, -\infty < x < \infty)$

$$u(x,0) = \begin{cases} 1, & x \le 0 \\ 1-x, & 0 \le x \le 1 \\ 0, & x \ge 1 \end{cases}$$

Find the solution.



(5) A spherical wave is a solution $u = u(r,t)$ of the 3-dimensional wave equation that depends only on the distance $r = \sqrt{x^2+y^2+z^2}$ and t. Then

$$u_{tt} = c^2\left(u_{rr} + \frac{2}{r}u_r\right)$$

- Change variables to $v = ru$. Thus

$$v_{tt} = c^2 v_{rr}$$

- Solve for v using d'Alembert's formula (!!!), as if it were one-dimensional
- Find the solution u with initial values

$$u(r,0) = \phi(r)$$
$$u_t(r,0) = \psi(r)$$

(One has to extend the functions as even ones: $\phi(-r) = \phi(r)$, $\psi(-r) = \psi(r)$.)

(Needless to say, Kirchhoff's formula <u>should</u> yield the same answer.)

(4)

$$\lim_{k\to\infty}\left\{k\int_{-\infty}^{\infty} e^{-(y-x)^2k^2}\frac{dy}{1+y^2}\right\} = ?$$