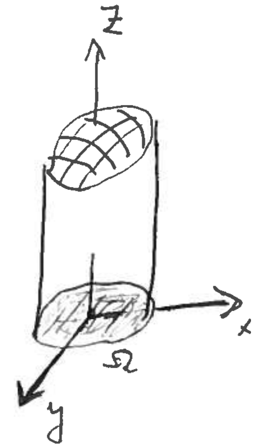
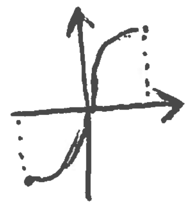
① Derive the Euler – Lagrange eqn for the problem of minimizing the area



$$A(v) = \iint\limits_{\Omega} \sqrt{1+|\nabla u|^2}\, dx\, dy$$

among all functions $z = u(x,y)$ with given boundary values on $\partial\Omega$, where Ω is a bounded domain.

② [Weierstraß's example]. Consider the variational integral



$$I(y) = \int_{-1}^{1} x^2 y'(x)^2\, dx$$

among all differentiable functions $y = y(x)$ that are continuous in $[-1, 1]$. Show that there is <u>no</u> minimizer with $y(-1) = -1$, $y(1) = 1$.

<u>Hint</u>:
$$y_k(x) = \frac{\overline{\text{arc}}\tan(kx)}{\overline{\text{arc}}\tan(kx)}, \quad k = 1, 2, 3, \ldots$$

③ Let $f \in L^2(\Omega)$, where Ω is bounded. Show that there exists a constant γ so that

$$J(v) \equiv \int_{\Omega} |\nabla v|^2 d\bar{x} - 2\int_{\Omega} f v\, d\bar{x} \geq -\gamma \|f\|^2_{L^2(\Omega)}$$

for all $v \in H^1_0(\Omega)$.

(4) Let Ω be a bounded domain. Find the Euler-Lagrange equation for the variational integral

$$I(u) = \int_\Omega [|\nabla u|^2 + e^u] \, d\bar{x},$$

when this is minimized among all functions in $C^2(\Omega) \cap C(\bar{\Omega})$ with given boundary values. Show that the extremal(s) cannot attain an interior maximum. Is the minimizer unique?