11. 1

11. 2

11. 4

11. 6

(1) Suppose that $\phi \in C^1(\Omega)$, $\Omega = (a, b) \subset \mathbb{R}$.

Show that

$$|\phi(x) - \phi(y)| \leq |x-y|^{1-\frac{1}{p}} \left\{ \int_a^b |\phi'(t)|^p dt \right\}^{\frac{1}{p}}, \quad p > 1.$$

(It follows that every $u \in W^{1,p}(\Omega)$ is equivalent to a <u>continuous</u> function satisfying

$$|u(x) - u(y)|^p \leq |x-y|^{p-1} \|u'\|_p^p \qquad (a < x < y < b)\Big)$$

(2) Prove that

$$\lim_{p \to \infty} \left\{ \int_a^b |\phi(x)|^p dx \right\}^{\frac{1}{p}} = \max_{a \leq x \leq b} |\phi(x)|$$

for every continuous function $\phi : [a, b] \longrightarrow \mathbb{R}$, where $b - a < \infty$.