Let $H = \{(x,t) \mid -\infty < x < \infty,\ t > 0\}$ denote the upper half-plane. Consider

$$(*) \quad \int_0^\infty \int_{-\infty}^\infty v[\phi_{tt} - c^2\phi_{xx}]\,dx\,dt = 0 \quad \text{for all } \phi \in C_0^\infty(H).$$

a) If $v \in C^2(H)$ satisfies (*), then

$$\frac{\partial^2 v}{\partial t^2} = c^2 \frac{\partial^2 v}{\partial x^2}.$$

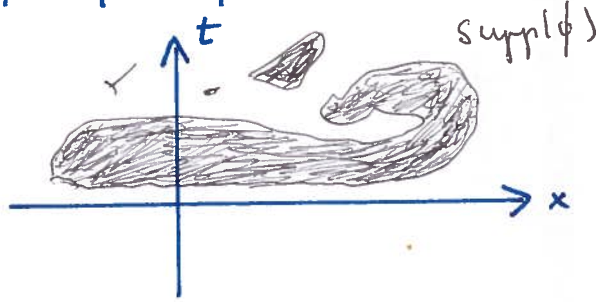
Prove this.

b) Show that, for example, $v(x,t) = |x - ct|$ satisfies (*). (It is a "weak solution" of the Wave Equation.)

closure.

$$\operatorname{supp}(\phi) = \overline{\{(x,t) \mid \phi(x,t) \neq 0\}} \subset\subset H$$

(The set where $\phi \neq 0$ does not touch the x-axis. In particular, $\phi_x, \phi_t, \phi_{xx}, \phi_{xt}, \dots$ are zero on the x-axis.)



9.8, 9.10, 9.11, 9.12