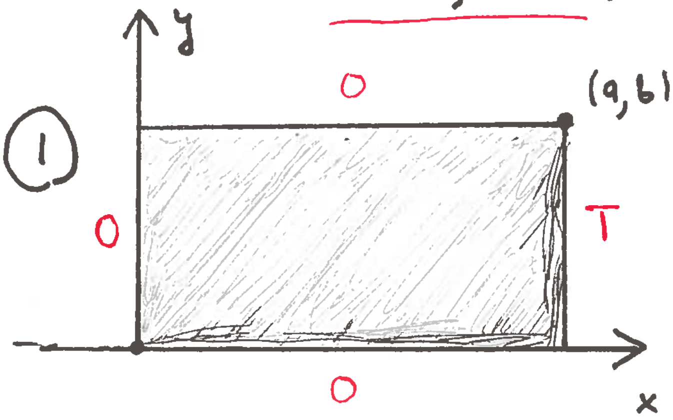
8.7, 9.1, and the following

① 

$0 \le x \le a,$

$0 \le y \le b.$

$$\Delta u \equiv \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Solve the Dirichlet problem for $\Delta u = 0$ in the rectangle with given constant boundary values 0, 0, 0, and T on the sides.

Answer:
$$u(x,y) = \frac{4T}{\pi} \sum_{\substack{n=1,3,5,\dots \\ \text{odd}}} \frac{\sinh\left(\frac{n\pi x}{b}\right) \cdot \sin\left(\frac{n\pi y}{b}\right)}{n \cdot \sinh\left(\frac{n\pi a}{b}\right)}$$

See YouTube
Unsteady Heat Equations in Rectangular Plates

② Assume that $g \in C(\mathbb{R})$ is bounded. Prove that

$$\lim_{y \to 0+} \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{g(t)}{(x-t)^2 + y^2}\, dt = g(x).$$

Remark: This is Poissons integral for the upper half-plane.

③ Verify
$$\nabla_{\bar{x}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{f(\bar{y})\, d\bar{y}^3}{|\bar{x} - \bar{y}|} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{\nabla_{\bar{y}} f(\bar{y})}{|\bar{x} - \bar{y}|}\, d\bar{y}^3$$

OBS!

for $f \in C_0^1(\mathbb{R}^3)$.