Exercises

I **Ex:** If $f = f(x)$ is continuous when $-1 \le x \le 1$, calculate

$$\frac{d^2}{dx^2}\left(\frac{1}{2}\int_{-1}^{1} |x-y|\, f(y)\, dy\right), \qquad -\infty < x < \infty .$$

Be careful, please.

II **Ex.** Is the parabolic maximum principle valid for the equation

$$u_t = x\, u_{xx}$$

in for example $-2 \le x \le 2$, $0 \le t \le 1$?

III **Ex.** Suppose that

$$u_t = u_{xx}$$

in $\mathcal{U} = \mathbb{R} \times (0,T)$. If $u \in C^2(\mathcal{U})$, $u \in C(\mathbb{R} \times [0,T])$ and $|u(x,t)| \le M < \infty$ in $\mathcal{U}$, prove that

$$u(x,t) \le \sup_{x \in \mathbb{R}} u(x,0), \qquad (x,t) \in \mathcal{U}.$$

Hint: $v = u(x,t) - \varepsilon(x^2 + 2t)$. Use the Max. Principle in $-L_\varepsilon \le x \le L_\varepsilon$, $0 \le t \le T$, where $L_\varepsilon \longrightarrow \infty$ is suitably chosen.

Solve

$$\begin{cases} u_y = \sin(u_x) \\ u(x,0) = \dfrac{\pi x}{4}, \quad -\infty < x < \infty \end{cases}$$

<u>Hint:</u> Taylor polynomial.

$$\begin{cases} u_t = k u_{xx} \quad (x > 0,\ t > 0) \\ u(x,0) = g(x), \quad x > 0 \\ u(0,t) = 0 \end{cases}$$

Find a formula for $u(x,t)$.