PARTIAL DIFFERENTIAL EQS 19. IX. 2016

(1) Consider

$$u_{zz} = u^2 u_x + u_{xy}^2$$

$$u(x,y,0) = x - y, \quad u_z(x,y,0) = \sin(x)$$

Find u_{xz}, u_{yz}, u_{zz} when $z = 0$. What about $u_{zzz}(x,y,0)$?

(2) Find a solution to the problem

$$\begin{cases} u_{xx} + u_{yy} = 0 \\ u(x,0) = 0 \\ u_y(x,0) = k^{-1}\sin(kx), \quad k > 0 \end{cases}$$

(Separate the variables). As $k \to 0$ the Cauchy data tend uniformly to 0. What happens to the corresponding solutions ? — The problem is not well-posed.

(3) Evaluate or simplify

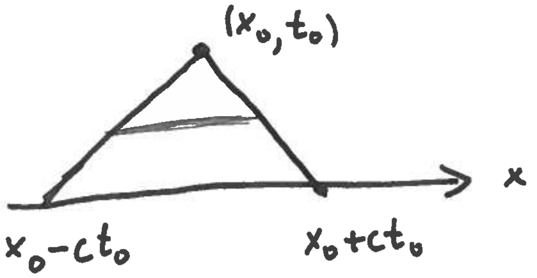
Recall $u_{tt} - c^2 u_{xx} = F(x,t)$.

$$u = \frac{1}{2c}\int_0^t d\tau \int_{x-c(t-\tau)}^{x+c(t-\tau)} F(\xi,\tau)\, d\xi$$

in the special case $F = F(t)$. (Compare with what the Ansatz $u = \phi(t)$, $\phi(0) = 0$, $\phi'(0) = 0$ yields. Is it the same ?)

(4) Suppose that $u_{tt} - c^2 u_{xx} = 0$ and that u has second derivatives up to the boundary in the triangle in the xt-plane.

Show that the "energy"



$$\mathcal{E}(t) = \frac{1}{2} \int_{x_0 - c(t_0 - t)}^{x_0 + c(t_0 - t)} (u_t^2 + c^2 u_x^2)\, dx \,, \qquad 0 \le t \le t_0$$

is decreasing. <u>Hint</u>: $\frac{d\mathcal{E}}{dt}$.

<u>Remark</u>: One can conclude that the initial values

$$u(x,0) = f(x), \quad u_t(x,0) = g(x)$$

have no influence of the value $u(x_0, t_0)$, when $x < x_0 - ct_0$ or $x > x_0 + ct_0$.