PARTIAL DIFFERENTIAL EQS 2016

① <u>Plane Waves</u> Find a condition on the vector $\bar{a} = (a_1, a_2, a_3)$ so that

$$u(\bar{x}, t) = F(\bar{a} \cdot \bar{x} - ct) \qquad (F \in C^2(\mathbb{R}))$$

is a solution of the eqn $u_{tt} = c^2 \Delta u$.

② <u>Radial Waves</u> Verify that

$$u(x,y,z,t) = \frac{F(r-ct)}{r} + \frac{G(r+ct)}{r}, \quad r = \sqrt{x^2+y^2+z^2}$$

formally satisfies the wave eqn.

③ Show that the potential

$$u = \frac{1}{r} e^{-\mu r}, \quad r = \sqrt{x^2+y^2+z^2}, \quad \mu = \text{Const.}$$

of the <u>strong</u> <u>nuclear</u> <u>force</u> satisfies the Yukawa equation

$$\Delta u = \mu^2 u \qquad (r > 0)$$

(actually, this is Helmholtz's eqn). Here $\mu > 0$.

④ Solve

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2} - c^2 \Delta u = 0 & \text{in } \mathbb{R}^3 \times (0, \infty) \\ u(x,y,z,0) = x^2 + y^2 \\ u_t(x,y,z,0) = 0 \end{cases}$$