COMPARISON PRINCIPLE FOR BOUNDED SOLUTIONS IN $\mathbb{R} \times (0,T)$

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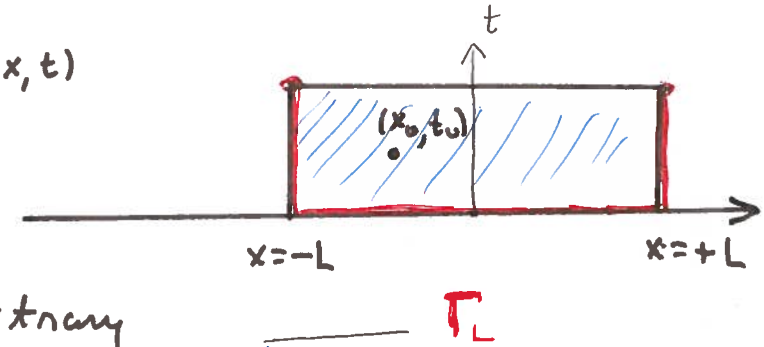
In the bounded domain $[-L, L] \times [0, T]$ the solution

$$v(x,t) = u(x,t) - \varepsilon(2tk + x^2)$$

satisfies the parabolic comparison principle:

$$v(x,t) \leq \max_{T_L} v(x,t)$$

Idea: $L \to \infty$.



Let (x_0, t_0) be an arbitrary point with $0 < t_0 < T$ and $|x_0| < L_\varepsilon$, when

By assumption $|u(x,t)| \leq M$.

$$L_\varepsilon = \sqrt{\frac{M}{\varepsilon}}$$ and $\varepsilon > 0$ small enough.

Adjusted in advance!

Remark: $L_\varepsilon \to \infty$ as $\varepsilon \to 0$.

$x = \pm L_\varepsilon$,
$$v(x,t) \leq M - \varepsilon(2tk + x^2)$$
$$\leq M - \varepsilon L_\varepsilon^2 = M - M = 0$$

$t = 0$
$$v(x,0) \leq u(x,0) \leq \sup_{-\infty < x < \infty} u(x,0)$$

Hence $v(x_0, t_0) \leq \sup\limits_{-\infty < x < \infty} u(x,0)$.

Thus
$$u(x_0, t_0) \leq v(x_0, t_0) + \varepsilon(2t_0 k + x_0^2)$$

As $\varepsilon \to 0$,
$$u(x_0, t_0) \leq v(x_0, t_0) \leq \sup_{-\infty < x < \infty} u(x,0)$$

— A similar proof goes for the minimum.