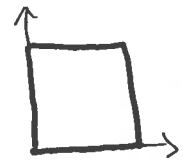


① Solve $\begin{cases} \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u^2 \\ u(0, y) = e^{-y^2} \end{cases}$ Where is the obtained solution valid?

② The variational integral

$$I(u) = \iint_{Q} (e^y u_x^2 + e^x u_y^2 - u) dx dy$$



is minimized among all functions with given boundary values on the sides of the square.

Find the differential equation (the EULER-LAGRANGE EQUATION) satisfied by the minimizer. - You may assume that all functions are sufficiently smooth.

HINT $I(u + \varepsilon \eta) \geq I(u)$, $\eta \in C_0^\infty(Q)$.

③ $\frac{\partial u}{\partial t} + \left(\frac{u^2}{2}\right)_x = 0$

Verify directly that

$$u(x, t) = \begin{cases} 1, & x < 0 \\ 0, & x > 0 \end{cases}$$

is not a weak solution.

$$\iint_{Q} (u \varphi_t + \frac{u^2}{2} \varphi_x) dx dt = 0$$

whenever $\varphi \in C_0^\infty(H)$

$$H = \{(x, t) \mid x \in \mathbb{R}, t > 0\}$$