P. Lq.

A variant of the proof of the parabolic MAXIMUM PRINCIPLE. Simple reasoning!

THEOREM Let $u \in C^2(\Omega \times (0,T))$, $u \in C(\overline{\Omega} \times [0,T])$. If $\Delta u \geq u_t$ in $\Omega \times (0,T)$, then the maximum of u is attained on the parabolic boundary, provided that Ω is bounded.

Proof: Let $\varepsilon > 0$ and consider

$$v = u - \frac{\varepsilon}{T-t}.$$

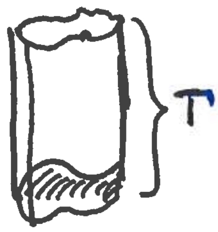
$$\Delta v = \Delta u, \qquad v_t = u_t - \frac{\varepsilon}{(T-t)^2}, \qquad v(x, T-) = -\infty$$

$$\Delta v \geq v_t + \frac{\varepsilon}{(T-t)^2} \geqslant v_t + \frac{\varepsilon}{T^2} > v_t$$

Therefore v cannot attain an interior maximum*) and the points when $t = T$ are out of the question. Thus $v(x,t) \leq \max_{\Gamma} v$, where Γ is the parabolic boundary. Thus

$$u(x,t) - \frac{\varepsilon}{T-t} \leq v(x,t) \leq \max_{\Gamma} v \leq \max_{\Gamma} u$$

$$u(x,t) \leq \max_{\Gamma} u + \frac{\varepsilon}{T-t}$$



Now, let $\varepsilon \longrightarrow 0+$ and conclude that $u(x,t) \leq \max_{\Gamma} u$.

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*) At an interior maximum point $\Delta v \leq 0$ and $v_t = 0$, by Calculus.

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