(1) Consider

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0 & (-\infty < x < \infty,\ t > 0) \\ u(x,0) = g(x) \\ u_t(x,0) = h(x) \end{cases}$$

Find a simple relation between $g(x)$ and $h(x)$ which produces a single wave-component travelling to the right.

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(2) The equation

$$a u_{xx} + b u_{xy} + c u_{yy} = f(x,y,u,u_x,u_y)$$

takes the form

$$A\bar{u}_{XX} + B\bar{u}_{XY} + C\bar{u}_{YY} = \text{lower order terms}$$

after the rotation

$$\begin{cases} X = x\cos\theta - y\sin\theta \\ Y = x\sin\theta + y\cos\theta \end{cases} \qquad (\theta = \text{const.})$$

of coördinates. Calculate $B^2 - 4AC$ in terms of a, b, c, θ, if a, b, c are constants.

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(3) The coefficients in

$$L(u) = a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial x \partial y} + c\frac{\partial^2 u}{\partial y^2}$$

are constants and $b^2 - 4ac = 0$ (parabolic). Find a linear transformation $\xi = \alpha x + \beta y$, $\eta = \gamma x + \delta y$ that transforms $L(u)$ into

$$L(u) = a\frac{\partial^2 \bar{u}}{\partial \eta^2} \qquad \left(\bar{u}(\xi,\eta) = u(\alpha x + \beta y, \gamma x + \delta y)\right)$$

(4) $i\Psi_t + \Psi_{xx} = 0$ Schrödinger eqn of a free particle

$$\Psi(x,t) = \int_{-\infty}^{\infty} \hat{\Psi}_0(k)\, e^{i(kx - k^2 t)}\, dk$$

Study the limit as $t \longrightarrow \infty$ with $c = \frac{x}{t}$ kept fixed. (Method of Stationary Phase is assumed to work well!)

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(5) Calculate $w(0,0,a,t)$ in 5.17 for a small a.

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(6) 5.13 b.