



The solution of the initial
value problem

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{1}{2} |\nabla u|^2 = 0 & \text{in } \mathbb{R}^n \times (0, \infty) \\ u(x, 0) = g(x) & \text{in } \mathbb{R}^n \times \{0\} \end{cases}$$

is given by the formula

$$u(x, t) = \min_y \left\{ g(y) + \frac{|x-y|^2}{2t} \right\}.$$

HOPF-LAX
FORMULA

Verify this for $g(x) = |x|^2 = (x_1^2 + x_2^2 + \dots + x_n^2)$

Remark You may take $n=1$.

