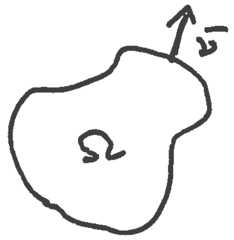
2014

① Prove the uniqueness of solutions to the Robin problem

$$\begin{cases} \Delta u = 0 & \text{in } \Omega \\ \dfrac{\partial u}{\partial \vec{\nu}} + a u = 0 & \text{on } \partial\Omega \end{cases}$$



where $a > 0$ is constant and Ω is a smooth bounded domain in $\mathbb{R}^3$. – What happens if $a = 0$?

② Is it possible that a solution u of problem ① is ≥ 0 in Ω.?

③ Solve the Heat Equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u = u(x,t)$$

in $(-\infty, \infty) \times (0, \infty)$ with inital values

$$u(x,0) = e^{-x}.$$

(Remark: Try to have a final answer without integrals)

④ $3u_y + u_{xy} = 0$

a) What is its type?

b) Find the general solution!

c) Discuss the Cauchy problem $u(x,0) = e^{-3x}$, $u_y(x,0) = 0$.

(5) A spherical wave is a solution $u = u(r, t)$ of the 3-dimensional wave equation that depends only on the distance $r = \sqrt{x^2+y^2+z^2}$ and t. Then

$$u_{tt} = c^2\left(u_{rr} + \frac{2}{r} u_r\right)$$

- Change variables to $v = ru$. Thus
$$v_{tt} = c^2 v_{rr}$$

- Solve for v using d'Alembert's formula (!!!), as if it were one-dimensional

- Find the solution u with initial values
$$u(r,0) = \phi(r)$$
$$u_t(r,0) = \psi(r)$$
(One has to extend the functions as even ones: $\phi(-r) = \phi(r)$, $\psi(-r) = \psi(r)$.)

(Needless to say, Kirchhoff's formula <u>should</u> yield the same answer.)