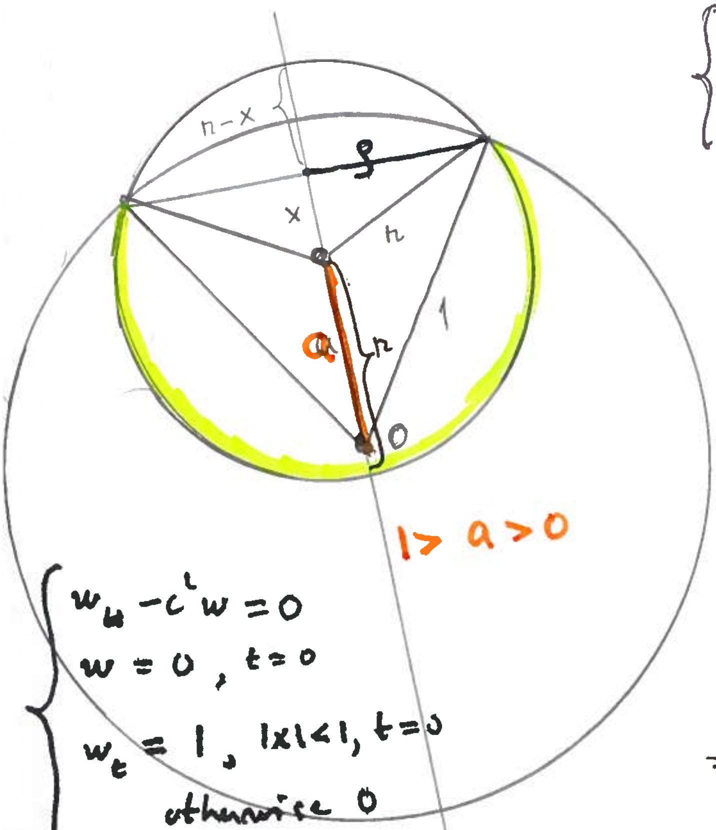
Addendum to 5.17
"FOCUSING EFFECT"



$1 > a > 0$

$$\begin{cases} w_{tt} - c^2 w = 0 \\ w = 0, \; t = 0 \\ w_t = 1, \; |x| < 1, \; t = 0 \\ \quad \text{otherwise } 0 \end{cases}$$

$$\boxed{r = ct}$$

$$\begin{cases} (r+x)(r-x) = \rho^2 & \text{cord theorem} \\ (a+x)^2 + \rho^2 = 1^2 & \text{Pythagoras} \end{cases}$$

$$x = \frac{1 - r^2 - a^2}{2a}$$

Area of the cap of hight $r + x$ is

$$2\pi r(r+x)$$

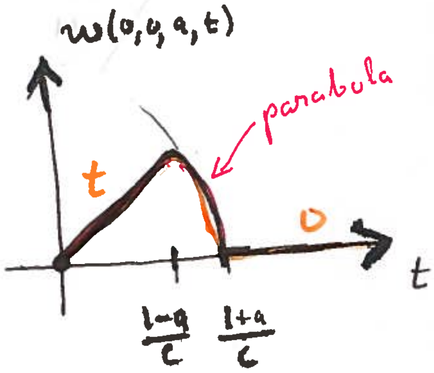
$$= 2\pi r\left(\frac{1 - r^2 - a^2 + 2ar}{2a}\right)$$

$$= \pi r\left(\frac{1 - (r+a)^2}{a}\right)$$

$$1 - a \leq ct \leq 1 + a$$

$$w(0,0,a,t) = t \cdot \frac{1}{4\pi (ct)^2} \cdot \pi ct \, \frac{1 - (ct+a)^2}{a}$$

$$= \frac{1}{4ca}\left[1 - (ct-a)^2\right]$$



$$w(0,0,a,t) = \begin{cases} t, & 0 \leq ct \leq 1-a \\ \dfrac{1-(ct-a)^2}{4ac} & 1 - a \leq ct \leq 1 + a \\ 0, & ct \geq 1 + a \end{cases}$$

The situation depends on a:
Notice that $u(0,0,a,t)$ is continuous, when $a > 0$,
but discontinuous at the focus $a = 0$ if $t = 1/c$