More specifics about importance sampling

Importance sampling is based on the use of a proposal distribution $g(x)$ from which it is easy to draw samples.

We can then express the expectation in the form of a finite sum over samples $x^{(l)}$ drawn from $g(x)$:

$$\begin{aligned} \mathrm{E}(p) &= \int p(z) f(z) dz \\ &= \int p(z) \frac{f(z)}{g(z)} g(z) dz \\ &\simeq \frac{1}{L} \sum_{l=1}^{L} \frac{f(x^{(l)})}{g(x^{(l)})} p(x^{(l)}). \end{aligned}$$

The quantities $w_l = f(x^{(l)})/g(x^{(l)})$ are known as importance weights. The importance weights correct the bias from a wrong distribution.

More specifics about importance sampling

That was a bit sloppy.

Importance sampling

We had

$$\mathrm{E}(p) = \int p(z) \frac{f(z)}{g(z)} g(z)\, dz.$$

Thus based on a random sample $x^{(1)}, \ldots, x^{(L)}$ from $g(x)$ the obvious Monte Carlo estimate is

$$\hat{\mathrm{E}}(p) = \frac{1}{\sum w_l} \sum_{l=1}^{L} p(x^{(l)}) w_l,$$

where $w_l = f(x^{(l)})/g(x^{l})$ are called importance weights.

Importance sampling II

It can be easily shown that the expected weight w_l equals one:

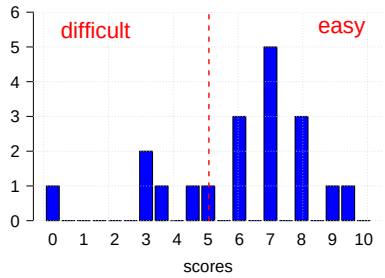
$$\mathrm{E}(w_l) = \int \frac{f(z)}{g(z)} g(z) dz = \int f(z) dz = 1\,,$$

so that $\sum_{l=1}^{L} w_l \approx L$. An alternative importance sampling estimate, replaces therefore $1/\sum_{l=1}^{L} w_l$ by $1/L$. The difference between these two estimates is usually small.

Feedback to first lecture block

Score from 0 to 10, where 0 means difficult and 10 easy.

Participiation 19 out of 34 ($\approx$ 56%) registered students.



- Difficult:

  Bayes (5), sampling algorithms (4),
  uninformative/Jeffreys priors (3), background (2)

- Easy:

  Distributions (2), sampling algorithms (3), Bayes