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Department of Mathematical
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TMA4295 Statistical
inference
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Exercise set 3

Read the questions carefully and make your own assumptions if needed.

1 Casella-Berger

⑥ 21, 39, 43

2 Bernoulli

Let x_1, \dots, x_n be a random sample from the Bernoulli distribution $B(p)$.

- a) Explain that this defines a statistical model. What is a natural choice for θ ? What is the model space Ω_Θ ? What is the data space Ω_X ?
- b) Show that this model is an exponential family. What is the natural statistic and the natural parameter?
- c) Show that the natural statistic is a minimal sufficient statistic. Is it complete?
- d) Illustrate the level set of the sufficient statistic and the conditional distribution in the case $n = 2$.
- e) How can the sufficiency principle be interpreted in terms of the level sets and the corresponding partition of the data space?
- f) Is the conditional distribution of the data given the sufficient statistic a uniform distribution?

3 The Basu theorem

State and prove the Basu theorem.

4 Exponential

Let the data x_1, \dots, x_n be a random sample from the Exponential distribution $\text{Exp}(\beta)$.

- a) Explain that this defines a statistical model. What is a natural choice for θ ? What is the model space Ω_Θ ? What is the data space Ω_X ?
- b) Show that this model is an exponential family. What is the natural statistic and the natural parameter?

- c) Show that the natural statistic is a minimal sufficient statistic. Is it complete?
- d) Find the conditional distribution of the data given the sufficient statistic.
- e) Illustrate the level set of the sufficient statistic and the conditional distribution in the case $n = 2$.
- f) Show that \bar{x} and $a = (x_i/\bar{x})$ are independent statistics using the Basu theorem.

5 Poisson

Let x_1, \dots, x_n be a random sample from the Poisson distribution $\text{Pois}(\mu)$.

- a) Explain that this defines a statistical model. What is a natural choice for θ ? What is the model space Ω_Θ ?
- b) Show that this model is an exponential family. What is the natural statistic and the natural parameter?
- c) Show that the natural statistic is a minimal sufficient statistic. Is it complete?
- d) Explain that there exists data such that the likelihood function from task **4** equals the likelihood function from the data here. What does the likelihood principle imply in this case?

6 Binomial

Let x_1, \dots, x_n be a random sample from the Binomial distribution $B(m, p)$ where both m and p are unknown.

- a) Explain that this defines a statistical model. What is a natural choice for θ ? What is the model space Ω_Θ ?
- b) Sketch the likelihood function for $n = 3$ and $x_1 = x_2 = x_3 = 3$
- c) Explain how to determine the maximum likelihood estimate.
- d) Is the MLE a sufficient statistic?
- e) Is this an exponential family?
- f) Determine a minimal sufficient statistic.

7 Casella-Berger

⑦ 5,6,10,15,17,50