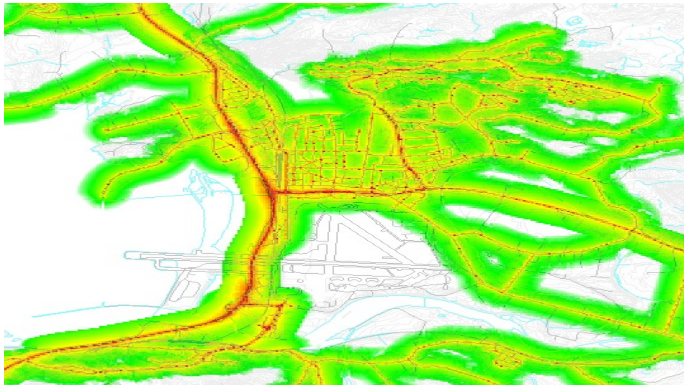


ISO Guide to the expression of Uncertainty in Measurements

Usikkerhetsberegninger og måledata (ISO GUM)

Gunnar Taraldsen

NTNU

April 22th 2022

The ISO GUMs main message

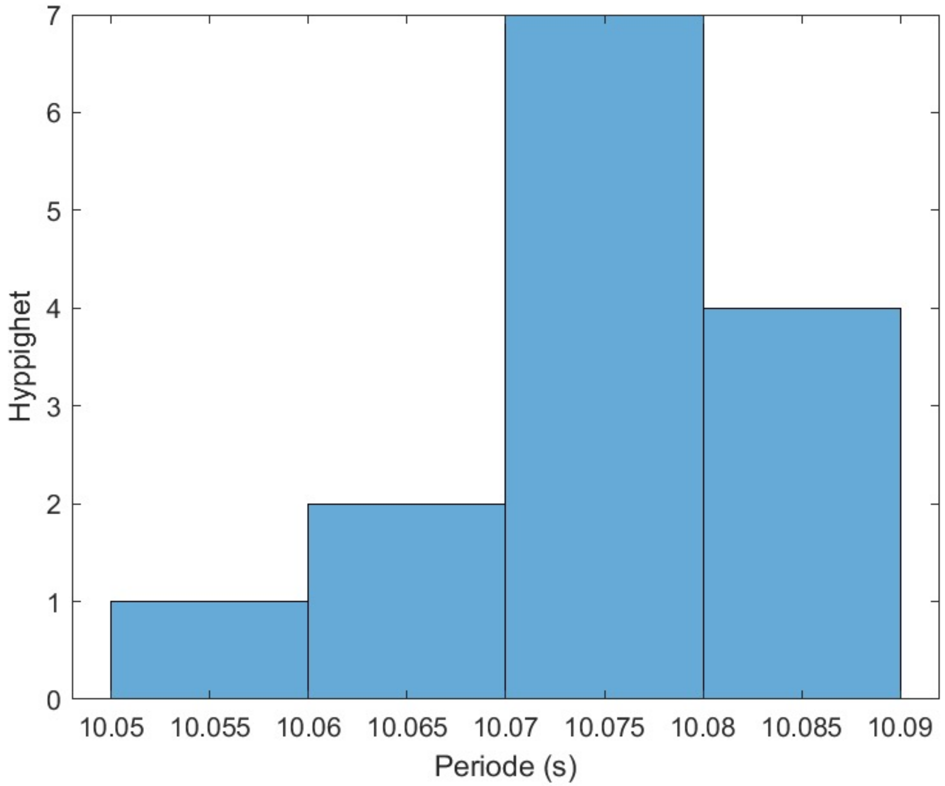
- Estimate
- Standard uncertainty
- Expanded uncertainty
- Credibility level

Pendel i realfagsbygget

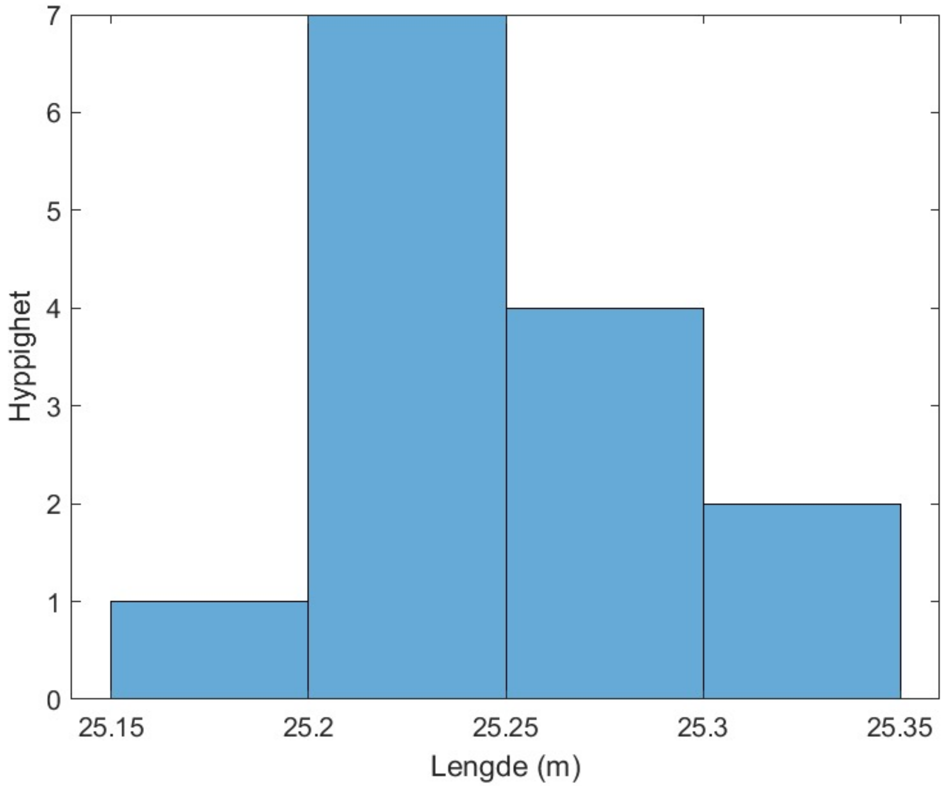
Tider på pendel

- aT1 = 10 + [57 66 69 71 72 73 74 74 75 78 80 82 87 88]'*1e-3;
- aT2 = 10 + [69 72 77 83 89 95]'*1e-3;
- aT = [aT1; aT2];
- nG = 9.82116199; % Uglaveien 76
- nG = 9.8214608; % NGU
- aL = nG*(aT/(2*pi)).^2;

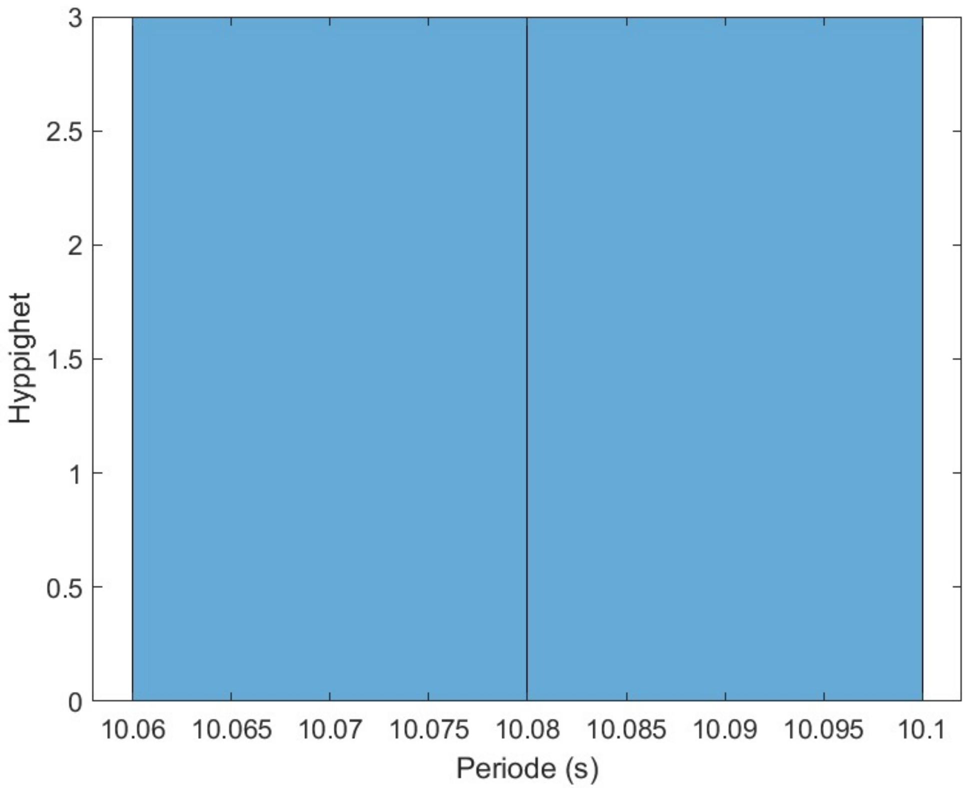
Studentmålinger



Studentmålinger

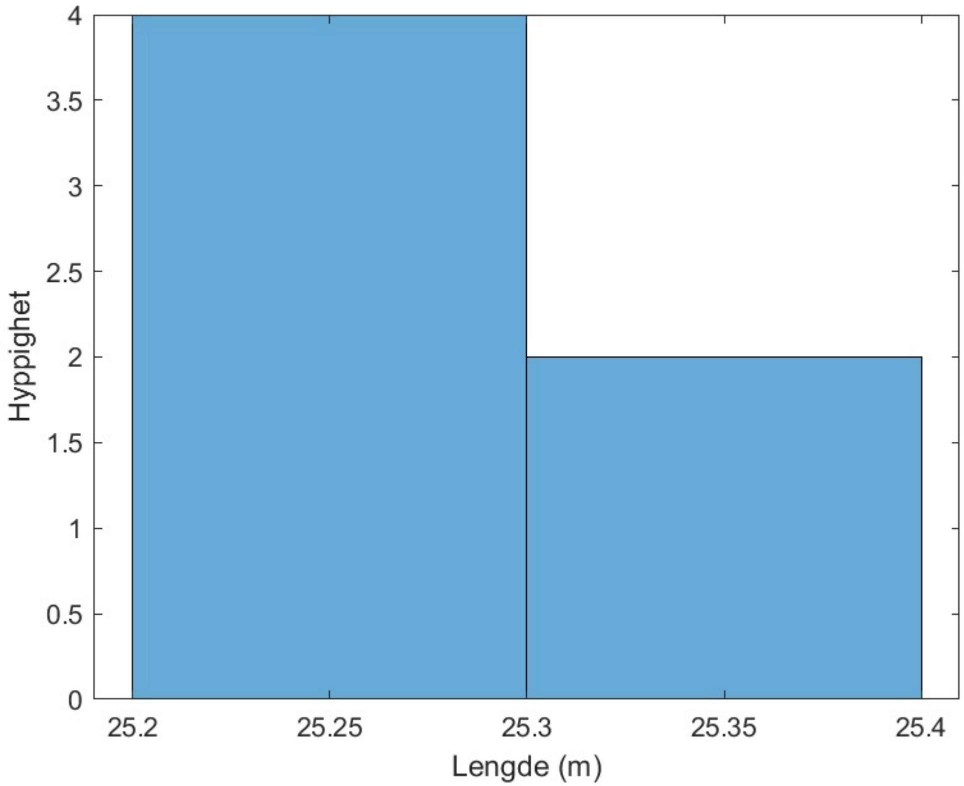


Mine

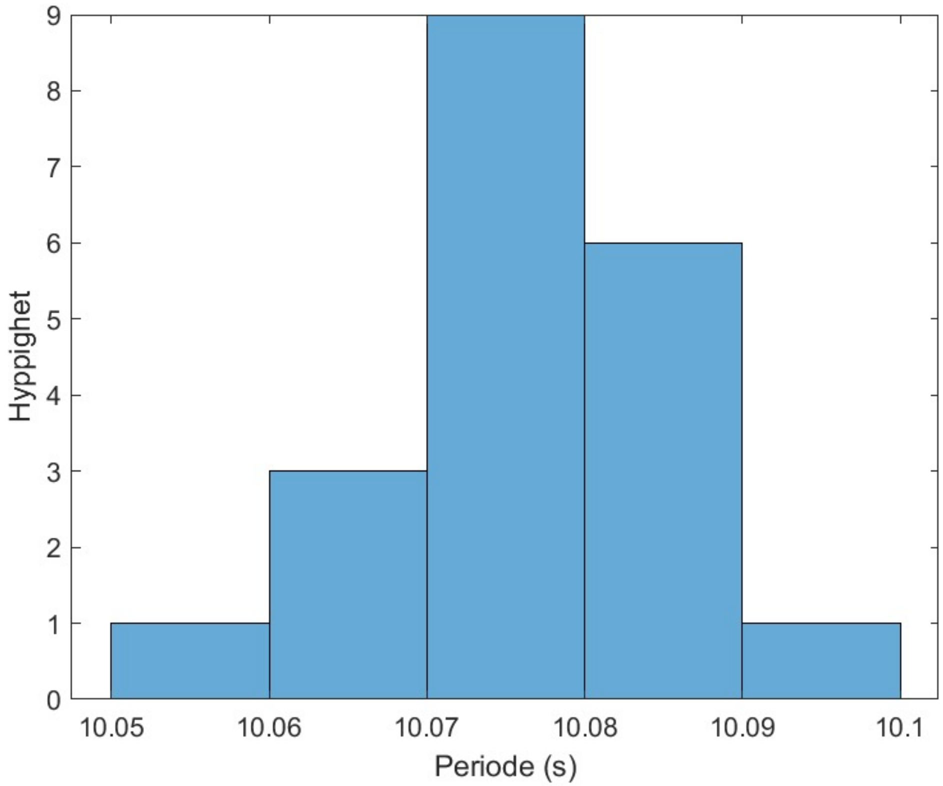


3
2.5
2
1.5
1
0.5
0
Hyppighet
10.06
10.065
10.07
10.075
10.08
10.085
10.09
10.095
10.1
Periode (s)

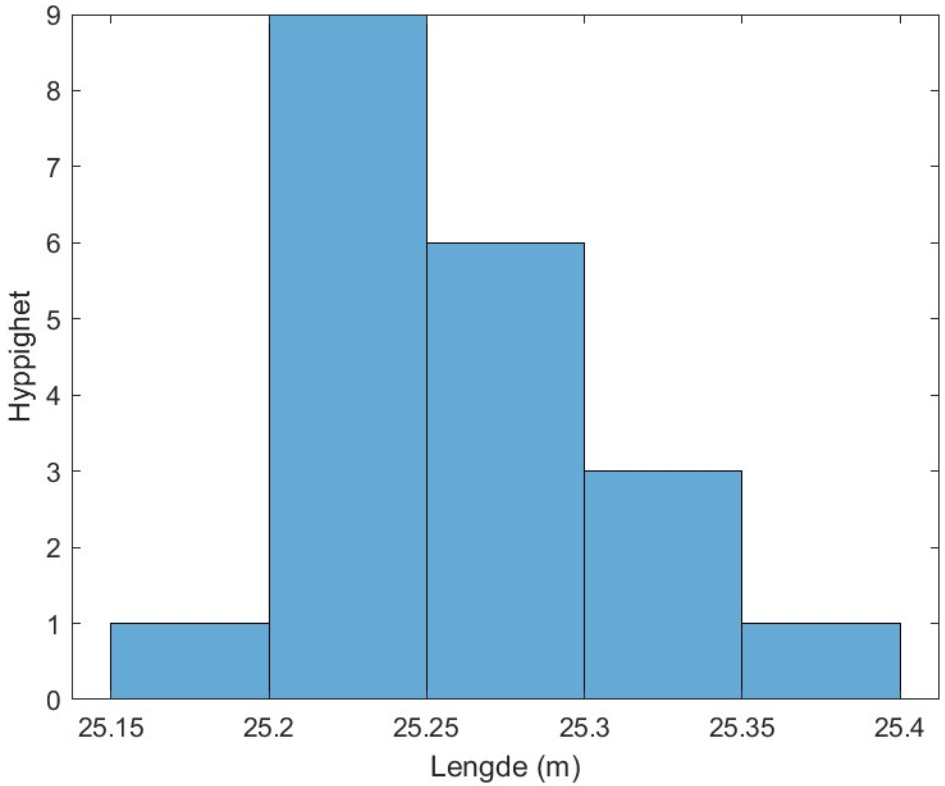
Mine



Alle


9
8
7
6
5
4
3
2
1
0
Hyppighet
10.05
10.06
10.07
10.08
10.09
10.1
Periode (s)

Alle



Studenter

- [nL, nStdErr, MUCI,SIGMACI] = normfit(lengthest(aT1, 9.8214608))

- nL = 25.2512
- nStdErr = 0.0409
- MUCI =
-     25.2276
-     25.2748
- SIGMACI =
-     0.0297
-     0.0659

GT

- [nL, nStdErr, MUCI,SIGMACI] = normfit(lengthest(aT2, 9.8214608))

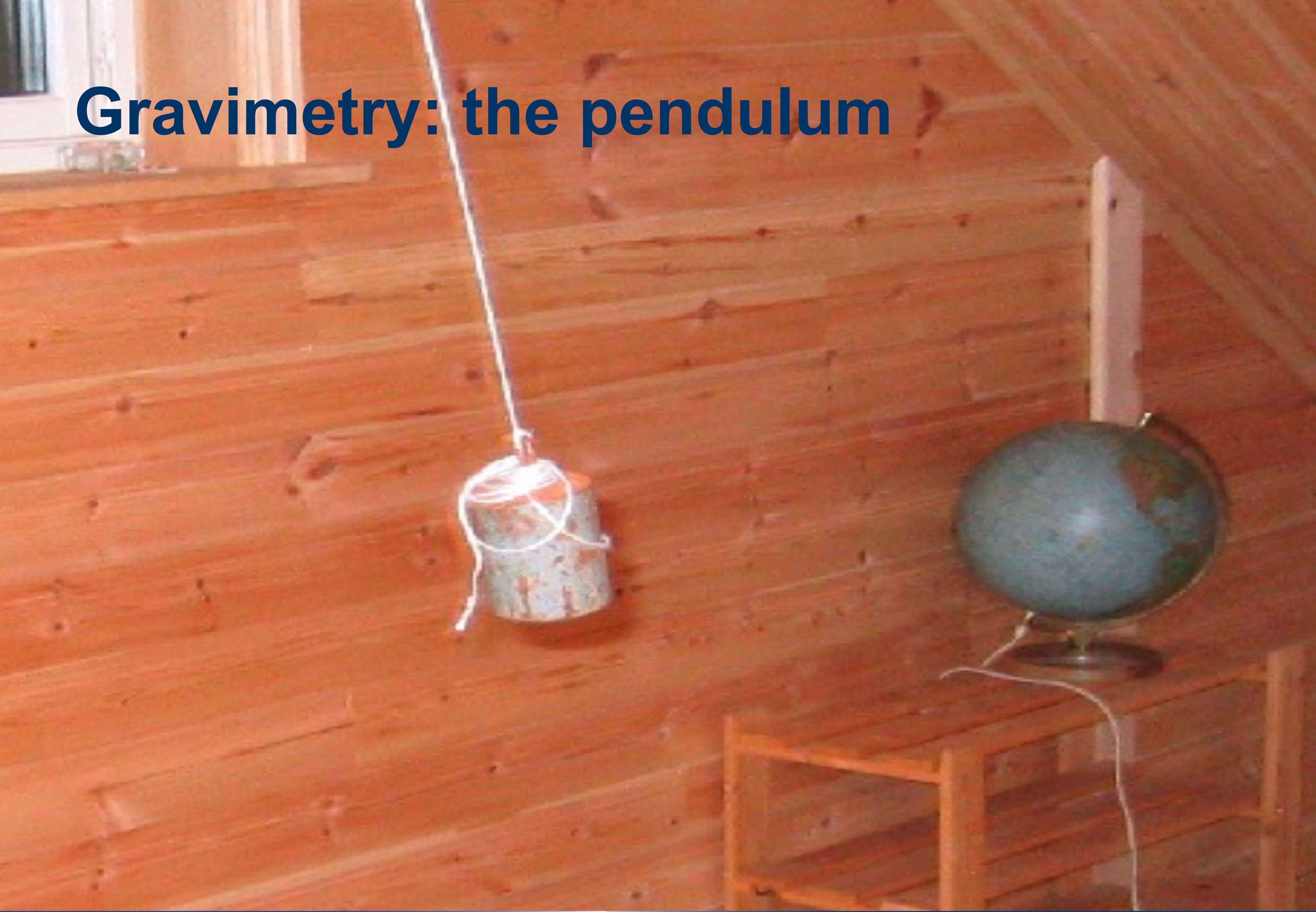
- nL = 25.2819
- nStdErr = 0.0504
- MUCI =
-    25.2290
-    25.3348
- SIGMACI =
-     0.0315
-     0.1236

Alle

- [nL, nStdErr, MUCI,SIGMACI] = normfit(lengthest(aT, 9.8214608))

- nL = 25.2604
- nStdErr = 0.0450
- MUCI =
- 25.2394
- 25.2815
- SIGMACI =
- 0.0342
- 0.0657

Gravimetry: the pendulum



The simple pendulum

- The acceleration **g** of gravity is given by

$$g = \left(\frac{2\pi}{T}\right)^2 l$$

where **T** is the period and **l** is the length of the pendulum.

- Measurement of g can hence be done by measurement of T and l.

- There are several possible estimation strategies, and some of them will be investigated.

Measurement of l

- The length was measured with manually with a measuring tape, and the result was

$$l = 366.7(1.0)\mathrm{cm}$$

where the number in parentheses is the numerical value of the expanded uncertainty *U(l)* referred to the corresponding last digits of the quoted result. This defines an interval estimated to have a level of confidence of 95 percent. The standard uncertainty is estimated to be u(l) = 0.5cm

- The uncertainty has been determined by a Type B evaluation.

Measurement of T

- The period was measured manually 10 times, and the result for an average of 10 periods was T = 3.8s +

$$\Delta T = [33, 41, 53, 41, 34, 52, 55, 34, 28, 34] ms$$

- Assuming a normal distribution, and application of the Student t distribution gives

$$T = 3.8405(69)\mathrm{s}$$

- The standard error is u (T) = 3.1 m s corresponding to a coverage factor k = 2.26 and 9 degrees of freedom.

Measurement of g

- The acceleration of gravity is

$$g = 9.8153(352)m/s^2$$

The ISO GUMs main message

- Estimate
- Standard uncertainty
- Expanded uncertainty
- Credibility level

References

It may be beneficial to consider (download your own private version from British standard http://www.bsonline.bsi-global.com)

[A] ISO, Guide to the expression of uncertainty in measurement (GUM)

[B] IS0 3534, 1985, Statistics - Vocabulary and symbols

[C] IS0 5725, 1994, Accuracy (trueness and precision) of measurement methods and results

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ISO and GUM [1, British foreword]

The International Organization for Standardization (ISO) requires that the 1993 edition of the Guide to the expression of uncertainty in measurement (GUM) be referenced when writing standards concerning the expression of uncertainty in measurement. The purpose of such guidance is:

(1) to promote full information on how uncertainty statements are arrived at;

(2) to provide a basis for the international comparison of measurement results.

ISO GUM introduction [1]

Just as the nearly universal use of the **International System of Units (SI)** has brought coherence to all scientific and technological measurements, a worldwide consensus on the evaluation and expression of uncertainty in measurement would permit the significance of a vast spectrum of measurement results in science, engineering, commerce, industry, and regulation to be readily understood and properly interpreted. In this era of the global marketplace, it is imperative that the method for evaluating and expressing uncertainty be uniform throughout the world so that measurements performed in different countries can be easily compared.

ISO GUM introduction [1]

- The ideal method for evaluating and expressing the uncertainty of the result of a measurement should be:
  - **universal**: the method should be applicable to all kinds of measurements and to all types of input data used in measurements; The actual quantity used to express uncertainty should be:
  - **internally consistent**: it should be directly derivable from the components that contribute to it, as well as independent of how these components are grouped and of the decomposition of the components into subcomponents;
  - **transferable**: it should be possible to use directly the uncertainty evaluated for one result as a component in evaluating the uncertainty of another measurement in which the first result is used.

The 8 step GUM procedure [1,8.1-8]

1. Define the measurand Y = f(X1, X2, …, XN)
2. Determine input quantity (xi)
3. Evaluate the standard uncertainty u(xi)
4. Evaluate the covariances
5. Calculate y = f(x1, x2, …, xN)
6. Evaluate the combined standard uncertainty uc(y)
7. Evaluate the expanded uncertainty U
8. Report y, uc(y), U(y) with level

- I would prefer u(y). The xi's and the y are conceptually similar.

Interpretation of the ISO GUM [1]

standard uncertainty

uncertainty of the result of a measurement expressed as a standard deviation

type A evaluation (of uncertainty)

method of evaluation of uncertainty by the statistical analysis of series of observations

type B evaluation (of uncertainty)

method of evaluation of uncertainty by means other than the statistical analysis of series of observations

A simple example [4,Blanquart, B. 2005a]

A simple example [4,Blanquart, B. 2005a]

- An operator has obtained 7 values of temperature in a water vessel, in repeatability conditions. The specifications of the thermometer are: "accuracy : +/- 0,5 °C"

- 32,5 °C 32,8 °C 32,1 °C
- 33,1 °C 32,7 °C 32,9 °C
- 32,3 °C

$$x_c = \bar{x}_{random} + C_{representativity} + C_{bias} + C_{displaying}$$

???

A simple example [4,Blanquart, B. 2005a]

$$u^2(x_c) = u^2(\bar{x}_{random}) + u^2(C_{representativity}) + u^2(C_{bias}) + u^2(C_{displaying})$$

$$u(\bar{x}) = \frac{\sigma}{\sqrt{n}} \qquad \frac{??}{??} \qquad \frac{0,5}{\sqrt{3}} \qquad \frac{0,05}{\sqrt{3}}$$

$$u^2(x_c) = \frac{0,35^2}{7} + \left(\frac{??}{??}\right)^2 + \left(\frac{0,5}{\sqrt{3}}\right)^2 + \left(\frac{0,05}{\sqrt{3}}\right)^2$$

A simple example [4,Blanquart, B. 2005a]

- Expression of the result

$$u(x_c) = 0{,}32$$

$$U(x_c) = 2 \times 0{,}32 = 0{,}64$$

- Temperature: 32,63 °C ± 0,64 °C (k=2) (95%)
  - unknown contribution of the homogeneity of the temperature in the vessel
  - major component: thermometer accuracy

A simple example [Blanquart, B. 2005a]: Interpretation

- The standard uncertainty u(y) is the ISO GUM notation for the square-root of the estimated variance of the estimator corresponding to y. This is an estimate of the standard deviation.
- The quantity y +- k u(y) defines an interval estimated to have a given level of confidence - typically 95%. The coverage factor k is usually in the range 2 to 3. The quantity U(y) = k u(y) is the expanded uncertainty.
- The following concepts are hence essential:
  - **Variance and standard deviation.**
  - **Estimator.**
  - **Estimate.**
  - **Level of confidence.**

Abstract of resolution paper

- A procedure is presented to evaluate the expanded uncertainty of a quantity about which discretized measurement data are available. The method is based on conventional statistics and depends on the value of the experimental variance and the resolution.
- The suggested procedure is compared with a recently suggested procedure based on Bayesian statistics.
- The ISO Guide to the expression of Uncertainty of Measurements (GUM) is discussed briefly. It is argued that both conventional and Bayesian statistics give a consistent interpretation of the GUM procedure, and the two approaches supplement each other.
- Conventional statistics estimates the uncertainty of the measurement procedure, while Bayesian statistics gives the uncertainty of the measurand.

Micrometer example

- **Let 7.489, 7.503, 7.433, 7.549, 7.526, 7.396, 7.543,7.509, 7.504, 7.383 be the result in mm of the measurement of some length μ with a micrometer.** [Lira, I: 2006]
- **Model**

$$X_i = \mu + \sigma Z_i$$

- **where**

$$Z_i \sim N(0,1)$$

are independent random variables.

- **The idealization is that the measurand μ is a property of the object, and σ is a property of the experiment as a whole.**

Micrometer example

A coverage level $p = 95\%$ gives the coverage factor $k_p = t_p(n-1) = 2.2622$. The estimate for μ is $\hat{\mu} = \overline{x} = 7.4835\,\text{mm}$, the standard error is $u(\mu) = s_x/\sqrt{n} = 0.0187\,\text{mm}$, and the expanded error is $U(\mu) = k_p u(\mu) = 0.0423\,\text{mm}$.

Micrometer example

The length is determined to be 7.483(42) mm, where the number in parentheses is the numerical value of the expanded uncertainty referred to the corresponding last digits of the quoted result. The coverage level is 95% and the coverage factor is 2.26.

Calliper example

Let 7.5 , 7.5 , 7.4 , 7.5 , 7.5 , 7.4 , 7.5 , 7.5 , 7.5 , 7.4 be the result in mm of a series of measurements of the same length μ with a caliper [21]. Assume that this can be modelled by independent random variables $Y_i \sim N(\mu, \sigma_Y^2)$.

Calliper example

The length is determined to be 7.47(3) mm, where the number in parentheses is the numerical value of the expanded uncertainty referred to the corresponding last digits of the quoted result. The coverage level is 95% and the coverage factor is 2.26.

Calliper and micrometer examples

The result $\mu = 7.47(3)\,\mathrm{mm}$ of the calliper measurement is more accurate than the result $\mu = 7.48(4)\,\mathrm{mm}$ of the micrometer measurement. The result is counterintuitive, but sometimes counterintuitive results are correct. In this particular case it is however the result of a too crude model.

Calliper and micrometer examples

The GUM, and common sense, demands that every relevant source of uncertainty shall be included in the model. The uncertainty due to instrument resolution has not been taken into account in the model. The calliper has a resolution $d = 100\,\mu\text{m}$ and the micrometer has a resolution $d = 1\,\mu\text{m}$.

Calliper and micrometer examples

A seemingly reasonable model for the calliper measurement is given by $Y_i = \mu + \sigma Z_i + dU_i$, where $Z_i \sim N(0,1)$ and $U_i \sim U(0,1)$. The uniform variable has $\mathrm{Var}(dU_i) = d^2/12$. The standard error $s_y/\sqrt{10} = 15.3\,\mu\mathrm{m}$, the standard error $d/\sqrt{12} = 28.9\,\mu\mathrm{m}$, and $u(\mu)^2 = s_y^2/10 + d^2/12$ give a combined standard error $u(\mu) = 32.7\,\mu\mathrm{m}$.

Calliper and micrometer examples

The micrometer measurement gives $s_x/\sqrt{10} = 18.7\,\mu$m, $d/\sqrt{12} = 0.3\,\mu$m, and a combined standard error $u(\mu) = 18.7\,\mu$m. In this case the instrument resolution is of no concern, and the micrometer result is more accurate than the calliper result.

Calliper and micrometer examples

There are several weak points in the argument given above. The model $Y_i = \mu + \sigma Z_i + dU_i$ fails to give realizations with values which are integer multiples of d. The S^2_{Y}/n is an unbiased estimator of the variance of the estimator $\overline{Y}$, so the above calculation of a combined standard error is dubious: *The effect of the resolution is already included in* S^2_{Y}/n.

Calliper and micrometer paper…

Calliper example

The length is determined to be 7.46(3) mm, where the number in parentheses is the numerical value of the Bayesian expanded uncertainty referred to in the corresponding last digit of the quoted result. The credibility level is 95% and the coverage factor is 2.0.

Micrometer example

The length is determined to be 7.48(4) mm, where the number in parentheses is the numerical value of the Bayesian expanded uncertainty referred to in the corresponding last digits of the quoted result. The credibility level is 95% and the coverage factor is 2.3.

The ISO GUMs main message

- Estimate
- Standard uncertainty
- Expanded uncertainty
- Credibility level