Norwegian University
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English

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EXAM IN COURSE TMA4240 STATISTICS

December 11th 2006
Time: 09:00–13:00

Permitted aids: Yellow A5 sheet with your own handwritten notes.
Tabeller og formler i statistikk, Tapir Forlag.
K. Rottmann: *Matematisk formelsamling*.
Calculator HP30S.

The examination results are due: January 10th 2007

Problem 1 The chocolate factory

A chocolate facotry wants to measure the porosity in the chocolate bars on the production line. The owner has aquired new measuring equipment. The equipment supplier states that the measuring equipment has normally distributed error with expectation $\mu = 0$ and variance $\sigma_m^2 = 0.0009$.

a) Assume that the stated measurement error is correct.

Compute the probability that the equipment measures a porosity that is higher than what the porosity really is.

Compute the probability that the measurement error is higher than 0.05.

If one takes two independent measurements with the equipment and uses the average of these as measurement, what is then the probability that the deviation between measured and true porosity is higher than 0.05?

The owner of the chocolate factory accepts $\mu = 0$, but doubts that the measurement variance from the supplier is correct. Consequently, he carries out five independent measurements on a reference chocolate bar with known porosity with an exact value of 0.15:

0.153 0.132 0.128 0.174 0.163

Disregard the round-off error in the fourth decimal of these measurements.

b) Write down the best unbiased estimator for the measurement variance based on the measurement values and assumptions above. State the reason for your choice.

Write down the distribution of the estimator, show that it is unbiased, and find the variance of the estimator.

Derive a 95% confidence interval for the correct measurement variance. Comment on the result.

Problem 2 Conflicts and conflikt management among students in relationships

In 2005 Mons Bendiksen from NTNU published a report with the title above, based on questionnaires distributed among NTNU students. The analysis includes $n = 418$ women and $m = 214$ men who lived in heterosexual relationships that had lasted three months or more. The complete report is published with permission on http://www.menn-mot-vold.no/.

We write K for a female respondent, $P(K) = \frac{418}{632}$ and accordingly M for male with $P(M) = \frac{214}{632}$. The probabilities relate to a random selection from the questionnaires.

One of the results was that 44.0% of the women and 27.2% of the men reported that they had practised psychological aggression against their partner during the last year. Denote this event by A. In other words, $P(A|M) = 0.272$ og $P(A|K) = 0.440$. In addition A^* denotes the complement of the event A.

Let B be the event that the partner has been psychologically aggressive. The report states that $P(A \cap B|K) = 29.2\%, P(A \cap B|M) = 21.6\%, P(A^* \cap B|K) = 14.8\%, P(A^* \cap B|M) = 5.6\%$.

a) Sketch the events A, B, M og K in a Venn-diagram, and shade or hatch $A \cap B$.

Compute the proportion of respondents who have practised psychological aggression that also are women, i.e. $P(A \cap K)$.

Compute the proportion of respondents who have exercised psychological aggression, $P(A)$.

b) Assume that we find a filled in questionnaire where we see that the respondent would be categorized as A. Compute the probability that the questionnaire has been filled in by a woman, i.e. $P(K|A)$.

Find an expression for the probability that a person who has been exposed to psychological aggression from the partner also has exercised psychological aggression against the partner, i.e. $P(A|B)$. Compute this probability.

We will from here on consider the event S : Exposed to sexual aggression from the partner during the last year.

Define X as the number of women reporting S, and Y as the number of men reporting S. The survey got the results $x = 38$ and $y = 10$, and it is stated that the difference is significant with a P-value less than 0.05.

c) Under which conditions is X binomially distributed?

Assume that the proportions from the survey equals the proportions in the whole population. From the same population we then randomly select five female students with male partner. Compute the probability that one or more of these five have been exposed to sexual aggression from their partner in the last year.

Let p_X be the proportion of female students in heterosexual relationships that have been exposed to S, and accordingly p_Y for the male students.

d) Explain why we can say that the estimators $\widehat{p}_X = X/n$ and $\widehat{p}_Y = Y/m$ are approximately normally distributed. Derive expressions for expectation value and variance for these two estimators.

Carry out the hypothesis test $H_0 : p_X = p_Y = p_0$ against the alternative hypothesis $H_1 : p_X \neq p_Y$ with significance level $\alpha = 0.05$.

Compute the P-value.

Problem 3 α-shifted exponential distribution

We will here consider a random variable X with probability density

$$f(x;\alpha,\beta) = \begin{cases} 0, & \text{for } x < \alpha, \\ \frac{1}{\beta}e^{-(x-\alpha)/\beta}, & \text{for } x \geq \alpha, \end{cases} \qquad \alpha > 0.$$

This can be referred to as an α-shifted exponential distribution. From this probability distribution we have n independent measurements $x_1, x_2, \ldots, x_n$.

In this problem you can use without proof that $\min(Z_1, Z_2, \ldots, Z_n)$ is exponentially distributed with parameter β/n if $Z_1, Z_2, \ldots, Z_n$ are independently identically exponentially distributed variables with parameter β.

a) Sketch the probability density $f(x;\alpha,\beta)$.

Assume that the parameter α is known. Write down the likelihood-function L and derive the maximum likelihood estimator (MLE) for β.

b) Assume that both parameters, α og β, are unknown.

Derive the maximum likelihood estimators for α and β. Check whether these estimators are unbiased or not. If not, show how the estimators can be adjusted so that unbiasedness is attained.