TMA 4230 - Exercises for April 8th, 2013

Exercise

Let $\mathcal{A}$ be a Banach algebra, and $\mathcal{B}$ be a Banach sub-algebra (that is a closed subspace, which is an algebra). Assume both $\mathcal{A}$ and $\mathcal{B}$ have a unit $1_{\mathcal{A}} = 1_{\mathcal{B}} = 1$.

Remember that λ is in the spectrum of $a \in \mathcal{A}$ if $a - \lambda$ is invertible in $\mathcal{A}$. This spectrum is noted $\sigma_{\mathcal{A}}(a)$. Similarly, $\sigma_{\mathcal{B}}(b)$ is the set of all μ such that $b - \mu$ is invertible in $\mathcal{B}$.

Let $b \in \mathcal{B}$. Then $b \in \mathcal{A}$.

- Show that $\sigma_{\mathcal{A}}(b) \subset \sigma_{\mathcal{B}}(b)$;
- Show that $\partial\sigma_{\mathcal{B}}(b) \subset \partial\sigma_{\mathcal{A}}(b)$, where ∂X is the topological boundary of a set (that is the closure of X minus the interior of X).
- What is the consequence for self-adjoint operators?