TMA 4230 - Exercises for March 4th, 2013

Exercise 1

Let A be a square invertible matrix. Show that there is a polynomial $P \in \mathbb{C}[X]$ such that

$$A^{-1} = P(A).$$

Exercise 2

Let A be a normal matrix, that is $AA^* = A^*A$, where $A^* = \overline{A^t}$. Let $\sigma(A)$ be the set of eigenvalues of A. It is admitted that there is an invertible matrix P such that

$$A = P^{-1}\mathrm{diag}(\lambda_1, \ldots, \lambda_n)P,$$

where $\mathrm{diag}(\lambda_1, \ldots, \lambda_n)$ is the diagonal matrix with entries λ_i on the diagonal.

Show that there is an isomorphism between $C(\sigma(A))$ and $\mathbb{C}[A, A^*]$, the algebra generated by A and A^*.

Exercise 3

Let $E = C([0, 1])$ and consider F the subspace of functions which are linear by parts. Show that F is dense is E.