Operators in Hilbert spaces

1. Hilbert-Schmidt operators (piece missed in a previous lecture)
2. Relation for adjoint operator.
3. Unitary operators:
   - Definition
   - Examples.
   - $U^* = U^{-1}$
   - Spectra of unitary operators
   - Eigenvectors of unitary operators.
4. Self-adjoint operators
   - Definition
   - Examples.
   - Taking adjoint operator as complex conjugation
   - "Real" and "imaginary" part of $A \in \mathcal{L}(H, H)$.
5. Quadratic form for self-adjoint operator. Norm evaluation.
6. Spectrum of selfadjoint operator.
   - Point spectrum is real
   - No residual spectra.
   - Invertibility condition: estimate from below.
   - Spectrum is real.
   - Spectral interval
   - Spectral radii for self-adjoint operator:
     $$r(A) = \|A\|.$$

7. Spectral theorem of self-adjoint compact operator.