Compact operators

1. Definition.
2. Example #1 : finite rank operators
3. Approximation by finite rank operators
4. Operator $T_a : \ell^p \to \ell^p$, where $a = (a_n)$

$$T_a : (\xi_k) \mapsto (a_k \xi_k)$$

5. Integral operators

5a. Green function for diff. eq-n.

6. Hilbert-Schmidt operators:
   - Definition, Hilbert-Schmidt norm
   - Independence of the choice of basis
   - Matrix representation
   - Integral HS operators

General properties of $\mathfrak{K}(X, Y)$

7. Closeness
8. Composition with bounded operators
9. Sum of operators
   (Minkowski sum of sets in X).
10. Algebraic digression: rings, ideals.

11. Adjoint operator is also compact.

12. $T \in \mathfrak{K}(X, Y)$, $\{x_n\} \subset X$, $x_n \xrightarrow{w} x \Rightarrow$

$$\Rightarrow Tx_n \xrightarrow{s} Tx$$

13. Remark: $T \in \mathcal{B}(X,Y) \Rightarrow \tilde{T} \in \mathcal{B}(X/_{\ker T} \to Y)$.

Fredholm.

14. <u>Reminder</u> from linear algebra.

15. <u>Statement</u> $T \in \mathcal{B}(X,Y) \Rightarrow \operatorname{Im}(T-I)$ is closed.

16. Fredholm alternative.

17. Application to integral operators