Three basic principle of functional analysis.

In this lecture I follow B&K Ch. 4.

- Hahn Banach theorem.
- Uniform boundedness
- Open mapping theorem.

Formulation of open mapping theorem. 1

Tool: Baire cathegory theorem.

- Reminder: - Metric space, complete metric space.
  Dense set.

  - $G_\delta$, $F_\sigma$ - sets.

  - Formulation of the theorem: $\{U_n\}$, $U_n$ - dense open $\Rightarrow \bigcap U_n$ dense.

- Corollary: $\{G_n\}$ - $G_\delta$ sets, dense $\Rightarrow \bigcap G_n$ - dense.

Example: $\mathbb{R} = \mathbb{Q} \cap (\mathbb{R} \setminus \mathbb{Q})$ - which of any is $G_\delta$.

- Proof of the Baire cathegory theorem.

- Why cathegory?
  Nowhere dense set. Examples.

- Set of 1st cathegory; set of 2nd cathegory.

Another formulation of Baire theorem:

Formulation of uniform boundedness theorem.

- proof:

Definition: Weak boundedness.

Corollary: Weak boundedness $\Rightarrow$ boundedness.

Banach - Steinhaus theorem.