

Hahn-Banach theorem

1. Preparation

- Partially ordered sets
- Examples.
- Upper bound, maximal element.
- Zorn lemma.

2. Formulation of Hahn-Banach theorem.

- ### 3. Proof:
- Prolongation from dense set
 - One-dimensional prolongation
 - Transfinite induction

4. Banach-limits

- ### 5. Corollary:
- Supporting functionals
 - Examples, uniqueness.
 - Functional separated points.
 - Linear span, completeness.

6. Second dual. Reflexive spaces. Examples.

- ### 7. Exercise:
- X is reflexive $\Leftrightarrow X^*$ is reflexive.

8. Adjoint operator:

- Reminder about \mathbb{C}^n
- Hilbert case.
- Banach case.

9. T -linear bounded $\Leftrightarrow T^*$ -linear bounded.

10. **Exercise:** $X \xrightarrow{T} Y \xrightarrow{S} Z$ hence $X^* \xleftarrow{T^*} Y^* \xleftarrow{S^*} Z^*$
prove $(ST)^* = T^*S^*$

11. Definition $\mathcal{L}(X, Y)$. This is a Banach space also.

12. $*$ is linear operator from $\mathcal{L}(X, Y)$ into $\mathcal{L}(Y^*, X^*)$.

13. Examples:

- Volterra.
- Hilbert Schmidt.

Operations with Banach spaces.

14. Direct sum

15. Quotient space.

16. Factorization by kernels of linear operators.

17. Annihilator of a set in X :

18. E -closed linear subspace in $X \Rightarrow$

$$E^* = X^* / E^\perp ; (X/E)^\perp = E^\perp$$