1) Let $X = C[0,1]$, $A: x(t) \mapsto \int_0^t x(\tau)\,d\tau$

Write explicite expression for A^n and verify $\|A^n\|^{1/n} \to 0$ as $n \to \infty$.

2) Let $T \in \mathcal{L}(X,X)$ and $T^{-1} \in \mathcal{L}(X,X)$ exists.

Prove that $\lambda \in \sigma(T) \Leftrightarrow \frac{1}{\lambda} \in \sigma(T^{-1})$.

3) Consider the right and left shifts in ℓ^2:

$R: (x_1, x_2, \dots) \mapsto (0, x_1, x_2, \dots)$; $L: (x_1, x_2, \dots) \mapsto (x_2, x_3, \dots)$

Let $\sigma_p, \sigma_c, \sigma_r$ denote point, continuous, and residual spectrum respectively.

Prove that $\sigma_p(R) = \emptyset$, $\sigma_c(R) = \{\lambda \in \mathbb{C}, |\lambda| = 1\}$,
$\sigma_r(\lambda) = \{\lambda \in \mathbb{C}, |\lambda| < 1\}$;

$\sigma_p(L) = \{\lambda \in \mathbb{C}, |\lambda| < 1\}$, $\sigma_c(L) = \{\lambda : |\lambda| = 1\}$;
$\sigma_r(L) = \emptyset$

4) Let $X = C[0,1]$; $T: x(t) \mapsto t\,x(t)$.
find $\sigma_p(T)$, $\sigma_c(T)$, $\sigma_r(T)$.

5) Let $T \subset \mathcal{L}(X,X)$ Prove that
$\sigma_r(T) \subseteq \sigma_p(T^*) \subseteq \sigma_r(T) \cup \sigma_p(T)$