Norges
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Institutt for matematiske fag

TMA4230
functional analysis
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Øving 3

Problem Set 3

$\boxed{1}$ Show that the dual of c_0 can be identified with ℓ^1.

$\boxed{2}$ For a Hilbert space $\mathcal{H}$ show that a bounded linear map $S : \mathcal{H} \to \mathcal{H}$ is an orthogonal projector if and only if $\|S^n x\| = \|x\|$ for all $n \in \mathbb{N}$ and $x \in \mathcal{H}$. Hint: Use the polarization identity.

$\boxed{3}$ For a Hilbert space $\mathcal{H}$ and an invertible, bounded linear map $S : \mathcal{H} \to \mathcal{H}$ and let L be a Banach limit on ℓ^∞. Show that $\langle x, y\rangle_L := L(\langle S^n x, S^n y\rangle)$ defines an innerproduct on $\mathcal{H}$.

$\boxed{4}$ Let X be a Banach space with bidual X^{**} and let $j : X \to X^{**}$ be the natural embedding. Show that (a) j is an injective bounded linear map. (b) Show that $j(X)$ is a closed subset of X^{**}.